SEMICONDUCTOR DETECTORS FOR
NEUTRON SPECTROMETRY AND MICRODOSIMETRY

Tesi di Dottorato di:
Ing. Andrea POLA

Relatori:
Prof. Stefano AGOSTEO
Prof. Alberto FAZZI

Tutor:
Prof. Stefano AGOSTEO

Coordinatore della Scuola di Dottorato:
Prof. Marzio MARSEGUERRA

XVIII CICLO
2003-2006
# CONTENTS

**Introduction** 1

I. *Neutron spectrometry: present status* 3
   I.1 Characteristics of neutron spectrometers 3
   I.2 A neutron spectrometer based on a silicon diode: feasibility study 5
   I.3 Fast neutron spectrometer based on silicon devices 8

II. *Microdosimetry and Solid-State Microdosimetry* 10
   II.1 Definition of microdosimetric quantities 10
   II.2 Experimental methods in microdosimetry 12
   II.3 Solid state microdosimeters based on silicon devices 14

III. *Analytical model of the response functions* 17
   III.1 Remarks and hypothesis 17
   III.2 Probability of neutron interaction in the radiator 19
   III.3 Equivalent dead layer 20
   III.4 The radiator-detector interface distribution 20
   III.5 Response functions (total and E stage) 25
   III.6 ΔE stage analysis 31
   III.7 ΔE – E scatter plot and bivariate distribution 38

IV. *Track length distributions and shape analysis* 45
   IV.1 Chord length distributions 46
   IV.2 Segment length distributions 49
   IV.3 ΔE stage track length distributions 53
   IV.4 Microdosimetric shape equivalence criteria 59
**Table of Contents**

V. **Characterization of the monolithic silicon telescope**

V.1 The monolithic silicon telescope: C-V measurements. .......................... 62
V.2 Noise characterization and remarks. .............................................. 67
V.3 Characterization of the sensitive thickness. ................................. 70
V.4 Ion beam analysis: charge collection and sensitive volume characterization. 77

VI. **A neutron spectrometer based on a monolithic silicon telescope**

VI.1 Experimental set-up ............................................................... 81
VI.2 Irradiations with monoenergetic neutrons. .................................. 83
VI.3 Unfolding algorithm. ................................................................. 88
VI.4 Continuous neutron spectra. .................................................... 89

VII. **A solid-state microdosimeter based on a monolithic silicon telescope**

VII.1 Main features of the detector ................................................ 96
VII.2 Field Funneling Effect. ......................................................... 97
VII.3 Monolithic Silicon Telescope as a Microdosimeter ...................... 99
VII.4 Experimental. ................................................................. 100
VII.5 Tissue-equivalence correction. ............................................. 101
VII.6 Shape analysis and correction criteria. ................................... 105
VII.7 Inter-comparison with a TEPC. ........................................... 106

**Conclusions** ............................................................... 108

**References** ............................................................... 110
INTRODUCTION

Semiconductor diode detectors are widely used in many radiation detection fields, especially in applications that require a low energy resolution. Since 1960, the technological development of devices employing semiconductor materials provided compact detectors with an effective thickness that can be varied to match the requirements of the intended application. Silicon predominates as the material for the manufacturing of the diode detectors to be used for charged particle spectrometry. Unlike germanium, it retains good energy resolutions at room temperature and can be employed to realize detection systems simple and cheap.

As described in chapter I, silicon pin diodes were recently investigated as neutron spectrometers. The detection of fast-neutron was realized by coupling the device with a polyethylene radiator and by unfolding the spectra of the recoil-protons generated by neutron fields. The detection system developed and tested were characterized by a detectable energy range mainly imposed by the photon background. This range of detection was limited in the interval (1.2-6 MeV), even by adopting pulse shape discrimination techniques.

In order to exploit the possibility of constructing devices with sensitive thickness of micrometric dimensions, silicon photodiodes were also investigated in microdosimetry applications (chapter II). The feasibility studies of a solid-state microdosimeter based on a pin diode highlighted some problems and limitations, such as the need of minimizing the so-called “field-funneling effect” (FFE), the non-tissue equivalence of silicon and the shape of the sensitive volumes.

The aim of this work is to investigate the use of an alternative silicon device, i.e. the monolithic silicon telescope, for both fast-neutron spectrometry and microdosimetry of neutron fields. This device, manufactured by ST-Microelectronics for nuclear physics applications, is characterized by two different stages: a thin $\Delta E$ stage (of the order of one $\mu$m in thickness) and a thick $E$ stage (about 500 $\mu$m thick). The $\Delta E$ stage is in principle a silicon microdosimeter when coupled to a TE converter. The $E$ stage can provide information on the energy and the type of the incident radiation. The ability of discriminating between different
types of charged particles could be exploited in order to realize a recoil-proton spectrometer with better discrimination performances and a lower detectable energy.

The study of this new detection system was faced both analytically and experimentally. As described in chapter III, a complete analytical description of the detector response to monoenergetic neutrons was developed and validated experimentally and with Monte Carlo simulations. Moreover, the problems related to the microdosimetric equivalence of sensitive volume shapes were faced by a thorough analysis about the track length distributions in the ∆E stage which is discussed in chapter IV.

An accurate experimental characterization of the depleted region and the sensitive thickness of the telescope stages was carried out in order to examine the behavior of the devices, especially of the ∆E stage, under irradiation with charged particles (chapter V).

The feasibility studies concerning with the neutron spectrometer and the silicon microdosimeter based on the monolithic silicon telescope were discussed in chapters VI and VII, respectively. The irradiations with mono-energetic and poly-energetic neutrons were performed at the Van De Graaff accelerator of the INFN-Laboratori Nazionali di Legnaro (Legnaro, Italy). The spectra obtained by unfolding the data collected with the spectrometer based on the silicon device were compared with the ones measured by time-of-flight techniques. The microdosimetric spectra measured with the monolithic silicon telescope were compared with those obtained by irradiating tissue equivalent proportional counter with a cylindrical sensitive volume.
Chapter I

Neutron Spectrometry: Present Status

Neutron spectrometry deals with the measurement of fast neutron spectral distributions. It is an important tool in several fields, such as nuclear physics and technology, fusion plasma diagnostics, radiotherapy and radiation protection. Different instruments and techniques have been developed to fulfill the requirements of the intended applications.

A thorough description and review about methods of neutron spectrometry can be found in references 1 and 2. Only a brief discussion about the characteristics and the applications will be presented in the following. The aim is to give the essential information to understand the advantages and limitations that involve the use of different types of spectrometers, especially at low neutron energies.

An overview about a feasibility study of neutron spectrometers based on different silicon detectors and detection systems will be also given.

I.1 Characteristics of neutron spectrometers

Neutron spectrometers based on pulsed-neutron methods will not be discussed in this survey, except for the time-of-flight technique that will be used as a reference in the investigation about the silicon spectrometer. A comparison between different types of neutron spectrometers is summarized in table 1 [2] with reference to features such as energy resolution, useful energy range and detection efficiency. Spectrometers 1-8 (table 1) collect all the data needed to determine the neutron energy in a single measurement, while the remainders (9-11) require a number of different detectors (9) or several measurements under different conditions or geometries (10 and 11). In applications such as nuclear physics or plasma diagnostic, energy resolution is usually of high priority, thus the spectrometers from number 1 to 8 are favored. In the fields of nuclear technology, radiation protection and
radiotherapy the detector should be simple to operate and the neutron fluence should be measured accurately over a wide energy range [2].

In the energy range from 50 keV to a few MeV, recoil-proton proportional counters are widely used. Their pulse height resolution is typically about 10 % FWHM for 1 MeV recoil ions. Their use above about 5 MeV is limited by wall and end-effects.

At neutron energies from 1 to 15 MeV, liquid (such as NE213) and plastic scintillators are commonly used with some limitations above 10 MeV [2]. They provide a response that is totally non-directional, they are quite inexpensive and can be produced in various sizes and shapes [1]. Pulse shape discrimination for γ-ray suppression is more effective with liquid scintillators. The pulse height resolution of an NE213 or BC501A neutron spectrometer was measured in ref. [3] and resulted to be of the order of a few percent in the energy interval 2-12 MeV. Moreover, with scintillator sizes of the order of a few cm (in diameter and length) they can offer a high detection efficiency (up to 20 %).

<table>
<thead>
<tr>
<th>Spectrometer</th>
<th>Typical characteristics for</th>
</tr>
</thead>
<tbody>
<tr>
<td>No.</td>
<td>Type</td>
</tr>
<tr>
<td>1</td>
<td>Recoil proportional counter</td>
</tr>
<tr>
<td>2</td>
<td>Organic scintillator</td>
</tr>
<tr>
<td>3</td>
<td>Recoil proton telescope</td>
</tr>
<tr>
<td>4</td>
<td>Capture-gated</td>
</tr>
<tr>
<td>5</td>
<td>(^3)He gridded ionization chamber</td>
</tr>
<tr>
<td>6</td>
<td>(^3)He-semiconductor sandwich</td>
</tr>
<tr>
<td>7</td>
<td>Diamond semiconductor</td>
</tr>
<tr>
<td>8</td>
<td>Time-of-flight</td>
</tr>
<tr>
<td>9</td>
<td>Foil radioactivation</td>
</tr>
<tr>
<td>10</td>
<td>Superheated drop (bubble)</td>
</tr>
<tr>
<td>11</td>
<td>Multisphere</td>
</tr>
</tbody>
</table>

\(^a\) Pulse height resolution
\(^b\) Energy resolution
\(^c\) Time-of-flight resolution

Table 1. Neutron spectrometers characteristics. Table taken from ref. 2.
I.2 A neutron spectrometer based on a silicon diode: feasibility study

Silicon devices are used in fast neutron spectrometry by measuring the recoil-protons generated in hydrogenated materials covering their surface. Diodes coupled with recoil-proton converters have also been proposed for personal neutron dosimetry [4-9]. These detectors should guarantee spatial and energy resolutions comparable to the ones provided by scintillators and can represent a valid alternative when a small detection system is needed. A neutron spectrometer based on a silicon device could be utilized in applications dealing with low-energy (a few MeV) neutron fields, like the ones which can be produced at facilities hosting low-energy ion-accelerators. In this context, such a spectrometer could be employed for assessing the neutron spectra in various radiation fields to be used, for instance, for irradiating biological cultures or for studying the radiation damage in electronic devices or in shielding calculation for radiation protection. Moreover, they could be attractive in those applications in which simplicity, compactness, cheapness, low-consumption and the possibility of realizing matrices of detectors are required.

The feasibility of a neutron spectrometer based on this detection technique was demonstrated in ref. 10. A windowless PIN diode (Hamamatsu S3509-06, thickness 0.5 mm) was coupled with a polyethylene converter (thickness 1 mm). The experimental verification of the response functions was performed at the Van De Graaff accelerator of the INFN-Legnaro National Laboratories (LNL, Italy) by irradiating the silicon device with monoenergetic neutrons generated by protons striking a thin LiF target (thickness 700 µg cm$^{-2}$). These

![Graph](image-url)

**Fig. 1** Spectrum of energy deposited in the silicon diode irradiated with 4.8 MeV monoenergetic neutrons. The spectrum is normalized to the unit neutron fluence. The distribution of recoil-protons is overwhelmed by the contribution of secondary electrons in the low energy part.
irradiations also allowed the energy calibration of the detector. A conventional electronic chain was used, consisting of a charge sensitive preamplifier, a spectroscopy amplifier and an ADC converter.

The minimum detectable energy for recoil-protons resulted to be set by energy deposition in silicon from secondary electrons produced in the detector assembly by gamma rays associated to the neutron field. This causes a tail in the low energy part of the deposited energy spectrum which overwhelms the distribution of recoil-protons (figure 1). In particular, it was found [10] that high-energy electrons traveling not perpendicularly to the diode surface are responsible for energy deposition from about 150 keV up to the minimum detectable energy for recoil-protons.

The minimum detectable energy of the spectrometer described in ref. 10 was about 1.5 MeV in an optimized set-up.

The maximum detectable energy was imposed by the thickness of the depleted layer at the operative bias voltage and was about 4.8 MeV for the utilized diode (depletion layer of about 163 µm thick at 15 V). The capability of resolving continuous neutron spectra was checked by measuring the energy distribution of secondary neutrons generated at 0° by 5 MeV protons striking a thick beryllium target. Also these measurements were performed at the LNL. The reconstructed spectrum is shown in fig. 2.

![Neutron yield vs Neutron energy](image)

**Fig. 2** Yield of neutrons generated at 0° by 5 MeV protons striking a thick beryllium target. The spectrum obtained unfolding the data with the spectrometer based on a silicon pin diode is compared with the one measured in ref [10].
The possibility of lowering the minimum detectable energy through pulse-shape discrimination (PSD) was investigated in ref. 11. In that work a new device based on a PIN diode in the “reverse-injection” configuration was characterized and tested. This “reverse-injection” configuration is realized by placing the polyethylene converter in contact with the $N^+$ layer (as suggested in refs. [12-14]). The signals from secondary electrons were discriminated from those due to low-energy recoil-protons by exploiting the different transit times of charge carriers generated in the silicon detector. The difference between the rise time of signals generated by electrons and recoil-protons were exploited by means of a bipolar shaping of a fast electronic chain.

The diode-spectrometer was irradiated with PSD and without PSD with neutrons generated by protons of several energies striking a thick beryllium target. The results of these measurements were compared with literature data measured with TOF techniques and are shown in fig. 3 at 5 MeV proton beam energies. At the highest neutron energies the agreement with all the spectra taken from the literature was fairly good. The new minimum detectable energy resulted to be about 1.2 MeV, while the maximum one was about 6 MeV. The detector sensitivity depends on the impinging neutron energy and is in the interval $4 \times 10^{-5} - 1.4 \times 10^{-3}$ counts per unit neutron fluence per unit sensitive area, for neutrons in the energy range 1.5-4.0 MeV.

Fig. 3 Yield of neutrons generated at 0° by 5 MeV protons striking a thick beryllium target. The spectrum obtained by unfolding the data with the spectrometer based on a silicon pin diode without PSD (green line) and with PSD (red line) is compared with the one measured in ref [11].
I.3 Fast neutron spectrometer based on silicon devices

A fast neutron spectrometer based on an integrated $\Delta E$-E silicon detector was studied in reference 15. A monolithic silicon telescope characterized by an epitaxial $\Delta E$ stage 10 $\mu$m thick and an E stage 200 $\mu$m thick was coupled to a thin polyethylene layer (about 28 $\mu$m thick). The aim was to realize a neutron spectrometer by measuring the energy and the emission angle of recoil-protons generated in the thin converter. According to the general theory of neutron elastic scattering, these two quantities are related to the incident neutron energy $E_n$ by the following formula:

$$E_n = \frac{E_p}{\cos^2 \theta}$$  \hspace{1cm} (1)

where $E_p$ is the initial energy of the recoil-proton and $\theta$ represent its emission angle. By assuming a polyethylene layer thickness infinitely thin, the energy $E_p$ and $\theta$ were calculated from the energy deposited in the two stages by the recoil-protons [15]. The incident neutron energy can be calculated directly with eq. (1) for each recoil event without employing unfolding techniques. The comparison between the calculated and the measured neutron spectrum from a RaD-Be source is shown in fig. 4. For 1.7 MeV neutrons, the detection efficiency and the energy resolution of this telescope spectrometer resulted to be about $10^{-4}$ and 31%, respectively. The investigated neutron energies were in the interval from 1.7 MeV and 15.1 MeV. Above about 5 MeV also the E stage behaves as a $\Delta E$ detector.

![Fig. 4](image.png)

**Fig. 4** Comparison between the calculated (line) and the measured neutron spectrum from a RaD-Be (histogram) [15].
It should be underlined that the hypothesis of an infinitely thin converter limits the value of the minimum detectable energy. At low neutron energies, below about 2 MeV, the range of recoil-protons is of the same order of the converter thickness and this hypothesis does not hold anymore. Moreover, at any neutron energy, the converter thickness cannot be neglected for those recoil-protons emitted at high angles. Therefore, this thickness should be taken into account for a more accurate description of the detector response function.

A feasibility study of a high-energy neutron spectrometer based on thick silicon detectors was discussed in ref. 16. The objective was to investigate the possibility of using silicon detectors for measuring the high-energy neutron component of cosmic radiation in thick structures, such as the International Space Station. This new method, based on the measurement of the detector response function to high-energy neutrons, aims to extend the measurement of neutron spectra from 12 MeV to 500 MeV. A commercial 500 µm thick silicon detector was irradiated with a neutron field produced by a pulsed 800 MeV proton beam bombarding a 7.5 cm thick tungsten target. The comparison with the differential neutron fluence measured with a fission chamber was satisfactory, as shown in fig. 5.

![Fig. 5](image_url)  
**Fig. 5** Comparison of measured differential neutron fluence from the fission foil chamber and the silicon detector for a 800 MeV proton beam bombarding a 7.5-cm-thick tungsten target [16].
Chapter II

MICRODOSIMETRY AND SOLID-STATE MICRODOSIMETRY

The term Microdosimetry refers to “a conceptual framework and corresponding experimental methods for the systematic analysis of the microscopic distribution of energy deposition in irradiated matter”. The objective of microdosimetry is “to develop concepts which relate some of the principle features of the absorption of ionizing radiation in matter to the size and perhaps the nature of the structure being affected” (International Commission on Radiation Units and Measurements, ICRU report 32 [17]). The limitation of the LET concept [17] lead to the formulation of a set of stochastic quantities which provide the fundamental basis for the description of the energy-deposition events in microscopic structures. The measurement of such quantities and their distributions in a well-defined volume is thus the aim of the experimental microdosimetric methods.

The applications of microdosimetric concepts concern with several fields, such as radiation biology, radiation chemistry, radiation protection, radiation therapy and dosimetry [18].

In this chapter the principal microdosimetric quantities are introduced also describing the typical experimental techniques developed for their measurement. In particular, applications of silicon devices as solid-state microdosimeters will be presented and discussed, since they are directly related to the main topic of this thesis, i.e the investigation of a new solid-state microdosimeter based on a monolithic silicon telescope.

II.1 Definition of microdosimetric quantities

The formal definitions of microdosimetric quantities are given in ICRU report 32 [17]. The elementary quantity is the energy deposit $\varepsilon_i$, which was introduced for the description of the inchoate spatial distribution of energy in charged-particle tracks. $\varepsilon_i$ is defined as the energy deposited in a single interaction $i$: 
\[ \varepsilon_i = T_{in} - T_{out} + Q_{\Delta m} \]  

(1)

where \(T_{in}\) and \(T_{out}\) are the energies of the incident ionizing particle and the sum of energies of all ionizing particles leaving the interaction (excluding the rest mass), respectively. \(Q_{\Delta m}\) are the changes of the rest mass energy of the atom and all particles involved in the reaction. \(\varepsilon_i\) is measured in joule or in eV. The energy imparted \(\varepsilon\) to the matter in a volume is the stochastic quantity

\[ \varepsilon = \sum \varepsilon_i \]  

(2)

where the summation is performed over all energy deposits \(\varepsilon_i\) in that volume, due to one or more energy deposition events. The unit of \(\varepsilon\) is the joule or eV. The quotient of \(\varepsilon\) by \(m\), where \(m\) is the mass of the matter in a volume, gives the specific (imparted) energy \(z\), that is:

\[ z = \frac{\varepsilon}{m} \]  

(3)

The unit of \(z\) is the joule per kilogram, namely the gray (Gy). \(z\) is a stochastic quantity characterized by a probability density distribution \(f(z)\). The expectation value, or mean specific energy,

\[ \bar{z} = \int_0^\infty z \cdot f(z)dz \]  

(4)

is a non-stochastic quantity and it is usually equal to absorbed dose \(D\). This depends on the fact that microdosimetry is typically concerned with volumes that are sufficiently small so that in most types of irradiation the dose can be considered constant [18]. Since \(z = D\), the integrand \(z \cdot f(z)\) in eq. 4 is the contribution to \(D\) delivered between \(z\) and \(z+dz\). Thus, it is useful to consider the so-called dose distribution of \(z\), \(d(z)\), given by:

\[ d(z) = \frac{z \cdot f(z)}{\bar{z}} \]  

(5).
By averaging $f(z)$ and $d(z)$ two different expectation values, $z_F$ and $z_D$, can be derived. They are defined as the frequency average and the dose average, respectively.

The quotient of $\varepsilon$ to $\bar{l}$, where $\bar{l}$ is the mean chord length in the considered volume, gives the lineal energy $y$:

$$y = \frac{\varepsilon}{\bar{l}}$$  \hspace{1cm} (6)

The unit of lineal energy $y$ is the joule per meter, but most commonly, the keV $\mu$m$^{-1}$. For a convex body, the mean chord length (defined as the mean length of randomly oriented chords in the volume), is given by Cauchy’s theorem, i.e. $\bar{l} = 4 \cdot V / A$, being $A$ the surface area of this body. $y$ is a stochastic quantity described by the probability density distribution $f(y)$. It is also useful to consider the dose distribution of $y$, $d(y)$, defined as the fraction of absorbed dose delivered with lineal energy within the interval $(y - y + dy)$, i.e.

$$d(y) = \frac{y \cdot f(y)}{\int_{0}^{\infty} y \cdot f(y) dy}$$  \hspace{1cm} (7)

The average of $f(y)$ and $d(y)$ give $\bar{y}_F$ and $\bar{y}_D$.

Since most microdosimetric distributions span a rather large spectrum of values, microdosimetric spectra are usually shown in a semi-logarithmic representation. In order to preserve the probabilistic meaning of the areas displayed in the spectra, a $y \cdot f(y)$ vs log$(y)$ representation is adopted. In fact:

$$\int_{y_1}^{y_2} f(y) dy = \ln 10 \int_{y_1}^{y_2} [y \cdot f(y)] d \log(y)$$  \hspace{1cm} (8)

and therefore the area delimited by $y_1$ and $y_2$ maintains the same information. Similarly, dose distributions are plotted by using a $y \cdot d(y)$ vs log$(y)$ representation.

### II.2 Experimental methods in microdosimetry

Microdosimetric measurements deal with the evaluation of experimental quantities closely related to the imparted energy $\varepsilon$. They are typically performed by exploiting the
principles of experimental simulation of microdosimetric volumes through proportional counters. The simulation of a microscopic volume of tissue of 1 g cm\(^{-3}\) is achieved by replacing it by a much larger cavity filled with a tissue-equivalent gas of much lower density. This requires the condition of equivalent energy loss for equivalent trajectories, that is:

\[
\Delta E_t = (S/\rho)_t \cdot \rho_t \cdot d_t = (S/\rho)_g \cdot \rho_g \cdot d_g = \Delta E_g
\]  

(9)

where \(\Delta E_t\) and \(\Delta E_g\) are the mean energy losses of a charged particle in a tissue of density \(\rho_t\) and in a gas of density \(\rho_g\), while \((S/\rho)_t\) and \((S/\rho)_g\) are the mass stopping powers. \(d_t\) and \(d_g\) are the track lengths in tissue and in the gas. The equivalence expressed by eq. 9 is verified if the atomic composition of tissue and gas are identical and the stopping powers are independent of the density.

The proportional counters conventionally applied in microdosimetry have walls made of tissue-equivalent (TE) plastic and are filled with tissue-equivalent gases (propane-based or methane-based). For ensuring that the secondary particle fluence is independent of density, the atomic composition of the wall and the gas must be identical, as stated by Fano’s theorem [19]. This condition cannot always be met in practice. Moreover, the requirement that the mass stopping powers are independent of density is not always fulfilled due to polarization effects in solids (with fast charged particles) [20].

However, tissue-equivalent proportional counters (TEPCs) are the most widely used microdosimeters. They consist of a spherical or a cylindrical gas chamber with a central anode wire electrically isolated from the surrounding chamber walls. The quality of a microdosimetric measurement assessed by a TEPC depends critically on the atomic composition of the filling gas. Any contamination should be avoided. This is particularly difficult, since the walls in tissue equivalent plastic adsorb the TE filling gas and release electronegative gases (including oxygen). Therefore, the best way of ensuring constant composition and pressure is to employ a gas flow system. However, to avoid complications and physical restrictions, counters are frequently employed in a sealed mode which allows operating for days or even weeks (with continuous maintenances).

The use of TEPCs for simulating microscopic tissue can lead to distortions of the experimental microdosimetric distribution due to the density difference between the cavity and the wall. These “wall-effects” are due to the fact that the energy is not deposited along straight lines because particles may scatter and their tracks may generate branches of
secondaries and tertiaries. The classification and the discussion about these effects can be found in ref. [17]. The errors introduced by these effects was estimated to be of the order of 10%. The minimization of the “wall-effect” errors was obtained by wall-less counters, by exploiting material grids or field-shaping electrodes to delineate the sensitive volume boundaries.

The characteristics of TEPCs are summarized in table 1 [21]. Presently, TEPCs represent the best detectors for microdosimetry because of their sensitivity to low energy particles, and tissue-equivalence. Moreover their sensitive volume is well defined and independent of the characteristics of the radiation field (i.e. particle LET, energy, etc.). Unfortunately, these detection systems are complex, expensive and with a poor spatial resolution.

<table>
<thead>
<tr>
<th>Category</th>
<th>Parameter</th>
<th>TEPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Detector performance</td>
<td>Energy Resolution</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>Low energy sensitivity</td>
<td>Excellent, Single ionizations, Minimum $y = 0.05 \text{keV/}\mu\text{m}$</td>
</tr>
<tr>
<td></td>
<td>Sensitive volume definition</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>Tissue Equivalence</td>
<td>Good</td>
</tr>
<tr>
<td></td>
<td>Radiation Hardness</td>
<td>Excellent</td>
</tr>
<tr>
<td></td>
<td>Spatial Resolution</td>
<td>Poor, 2.5 cm, 0.5 mm best case [72]</td>
</tr>
<tr>
<td></td>
<td>Wall Effect Immunity</td>
<td>Poor</td>
</tr>
<tr>
<td></td>
<td>Model cell array</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Shape design flexibility</td>
<td>Moderate</td>
</tr>
<tr>
<td>Ease of use</td>
<td>Calibration</td>
<td>Simple</td>
</tr>
<tr>
<td></td>
<td>Cost</td>
<td>High</td>
</tr>
<tr>
<td></td>
<td>Portability</td>
<td>Moderate</td>
</tr>
<tr>
<td></td>
<td>System Complexity</td>
<td>Poor: requires HV supply and gas supply</td>
</tr>
<tr>
<td></td>
<td>In-vivo use</td>
<td>No</td>
</tr>
<tr>
<td></td>
<td>Integration</td>
<td>Poor</td>
</tr>
</tbody>
</table>

Table 1 Main characteristics of Tissue Equivalence Proportional Counter (TEPC) [21]. The shaded areas identify the best performance of the device for each parameter.

II.3 Solid state microdosimeters based on silicon devices

The use of semiconductor devices for microdosimetry has been investigated since 1980 [22]. The first device employed for this purpose was a 7 $\mu\text{m}$ thick Si(Li) detector irradiated with a beam of negative pions. The results of these measurements were compared to the data acquired with a tissue-equivalent proportional counter (TEPC) simulating a 2 $\mu\text{m}$ thick site. Significant discrepancies were observed, mainly due to the dimensions of the sensitive volume of the silicon detector. Since then, several devices (mainly diodes) coupled
to tissue-equivalent converters were employed for silicon microdosimetry [23-26] of neutron fields. Also in those works, the differences from the lineal energy spectra measured with TEPCs were mainly ascribed to the shape and the dimensions of the sensitive volumes.

The most attractive feature of a silicon microdosimeter is the possibility of constructing devices of micrometric dimensions. This allows to measure physical events in a real micrometric site differently from TEPCs where the latter is simulated acting on the gas pressure. Anyway, there are some problems concerning with the use of silicon devices: (i) silicon is not a tissue-equivalent material, especially for neutrons. All secondaries produced by neutrons interacting directly with silicon should be discarded or, at least, they should give a negligible contribution with respect to those generated in the tissue-equivalent converter; (ii) the “field-funnelling effect” which leads to a dependence of the thickness of the zone useful for charge collection on the LET of the impinging particle; (iii) the electronic noise which imposes the minimum detectable energy; (iv) the shape and the dimensions of the sensitive zone (i.e. the depletion layer).

Among the items listed above, the “field-funnelling effect” [27] plays a major role. This effect is due to a local distortion of the electric field in the depletion layer, induced by high-LET particles, leading to the collection of electron-hole pairs produced in the non-depleted zone. This effect is responsible for the dependence on LET of the thickness of the sensitive zone and therefore it must not be present in a device applicable for microdosimetry, where this thickness has to be known accurately.

The field-funnelling effect was observed in PIN diodes [28]. Fig. 1 shows the results of the irradiation with monoenergetic neutrons of a device consisting of a matrix of nine n-p diodes (each one with a sensitive area of 1 mm$^2$) which was realized through the AMS BiCMOS 0.8 $\mu$m technology. The thickness of the depletion layer was assessed with capacitance measurements and resulted to be less than 2 $\mu$m. The edge of the spectrum of the deposited energy shifts towards higher values while increasing the energy of the incident neutrons. Therefore, the useful thickness for charge collection is higher than that of the depletion layer. Neutrons can transfer up to all their energy to recoil-protons in an elastic collision. Moreover, in such a knock-on reaction, recoil-protons have the same direction of the incident neutrons. Since monoenergetic neutrons were directed normally to the converter-detector surface, the shift of the recoil-proton edge could not be ascribed to tracks of maximum transferred energy travelling not perpendicularly to the detector surface. Therefore only the field-funnelling effect can be responsible for this shift. Recoil-protons of maximum transferred energy are characterized by a range in silicon which is higher than the nominal
thickness of the depletion layer for all the considered neutron energies. As an example, the range in silicon of 0.573 MeV protons (corresponding to the lower neutron energies considered in these irradiations) is 7 µm which is higher than the thickness of the passivated and the depletion layer.

In general, the n-p diodes cannot be considered for silicon microdosimetry.

More recently, a silicon microdosimeter consisting of an array of microscopic pn junctions based on the silicon-on-insulator (SOI) technology was realised and tested with various radiation fields for hadrontherapy [29, 30]. The performance of semiconductors as microdosimeters was thoroughly discussed by Bradley [21]. Guidelines for the realization of these devices are given in that work, together with a theoretical approach to correct the discrepancies due to the shape of the sensitive volume and tissue-equivalency.

![Graph](image)

Fig.1 Results of the irradiation with monoenergetic neutrons of energy $E_n$ of a device consisting of a matrix of nine n-p diodes (sensitive area of 1 mm$^2$) which was realized through the AMS BiCMOS 0.8 µm technology [28].
Chapter III

Analytical Model of the Response Functions

A complete analytical description of the response to monoenergetic neutrons of the monolithic silicon telescope represents a fundamental step for the analysis of the experimental data. The knowledge of the analytical response of both the E and the ΔE stage allows to optimize the unfolding procedure for the reconstruction of the impinging neutron spectrum. Moreover, the calculation of the track length distributions of secondary particles in the ΔE stage permits to determine the correction functions for the measured microdosimetric spectrum.

It should be underlined that the theoretical model described in the following is based on guidelines given by Foglio Para and Agosteo in ref. [31]. The analytical procedure described in ref.[31] is based on a simple, but approximated, range-energy relation deduced by fitting the data obtained with the SRIM code [32, 33] above 1 MeV. Since the aim of this study is to decrease the minimum detectable energy, it is necessary to develop a more general model.

III.1. Remarks and hypothesis

The detection system consists of a silicon telescope coupled to a polyethylene layer 1 mm thick. The silicon device (fig. 1) is characterized by a dead layer of titanium (about 0.24 μm thick), a thin ΔE stage (about 2 μm thick), and a 500 μm thick E stage.

The spectra measured by the telescope consist of the energy distribution of the secondary charge particles generated in the radiator by the neutron field via elastic scattering in the polyethylene radiator, (CH₂)n. Generally, for a neutron of energy Eₙ, the energy transferred to a nucleus of mass A through an elastic collision is given by:

\[ E_{RN} = \frac{4A}{(A+1)^2} \cdot \cos^2 \theta \cdot Eₙ \]  

(1)
where $\theta$ is the scattering angle of the recoil-nucleus with respect to the neutron direction of incidence in the laboratory system. The hydrogen nuclei can recoil with an energy in the interval from 0 to $E_n$, while carbon gains $0.284 \cdot E_n$ at maximum. Thus, for the neutron energies investigated in this work (< 8 MeV), only a small fraction of recoil-carbons can reach the detector, because their range in polyethylene is shorter than that of recoil-protons at the same neutron energy. Moreover, the atomic density (atoms per cm$^3$) and the scattering cross section of carbon is on average lower than that of hydrogen. For these reasons, the contribution of carbon atoms to the response function will be neglected.

![A schematic view of the ST-Microelectronics Monolithic Silicon Telescope](image)

**Fig. 1** A schematic view of the ST-Microelectronics Monolithic Silicon Telescope

The maximum detectable energy corresponds to the highest energy of recoil-protons stopping completely inside the silicon detector. This energy is about 8 MeV and it is imposed by the thickness of the telescope (about 500 µm). By contrast, the range of 8 MeV protons in polyethylene is about 800 µm, which is lower than the radiator thickness. Only recoil-protons starting from the lower part of the radiator (referred in the following as “polyethylene layer of interest”) can reach the sensitive volume of the detector. The total radiator layer (1 mm thick) is thus equivalent to an “infinite” thickness.

Below 10 MeV, only s-wave neutrons can interact with the nucleus, and the scattering angular distribution can be considered isotropic in the center-of-mass system (the deviation from isotropy is lower than 6%). In the laboratory system, recoil-protons are distributed uniformly from $\theta = 0^\circ$ to $\theta = 90^\circ$ in each $[d \cos^2 \theta]$ element. Since the recoil-proton energy $E_p$,
is given by $E_n \cos^2 \theta$ (from equation (1)), the uniform distribution of $\cos^2(\theta)$ corresponds to the uniform distribution of $E_p$.

It should be underlined that the analytical approach described in the following is based on the energy-range and energy-stopping power relations taken from ICRU report n 49 [34].

III.2. Probability of neutron interaction in the radiator

The analytical response functions described in the following are calculated by normalizing the distributions to a single neutron interaction. Therefore, for a quantitative evaluation of the fluence response functions, these distributions must be multiplied by the probability $\pi(E_n)$ of generating a recoil-proton in the polyethylene radiator per unit neutron fluence. This probability must take into account the attenuation of the neutron field inside polyethylene, which increases for decreasing neutron energies. The attenuation of the neutron fluence in the polyethylene layer at different neutron energies is shown in fig. 2. Value higher than 4 % can be observed below 1 MeV.

![Fig. 2](image.png)

**Fig. 2** Relative fluence attenuation in polyethylene (1 mm thick). The cross sections are taken from JEF 2.2 [35].

The probability $\pi(E_n)$ was evaluated according to an exponential attenuation of the neutron fluence and neglecting multiple scattering. By integrating the probability of undergoing an interaction with hydrogen nuclei per unit path length over the layer of interest, that is:
\[
\pi(E_n) = A \cdot \int_{L-R^{\text{poly}}(E_n)}^{L} e^{-\Sigma_{\text{tot}}x} \cdot \Sigma_H \cdot dx
\]  

(2)

where \(A\) is the detector sensitive area, \(L\) is the thickness of the polyethylene layer (1 mm), \(\Sigma_H\) is the macroscopic cross section of hydrogen and \(\Sigma_{\text{tot}}\) is the total macroscopic cross section (hydrogen plus carbon) at the energy \(E_n\) and \(R^{\text{poly}}(E_n)\) is the maximum range in polyethylene of recoil-protons, i.e. the thickness of the layer of interest.

III.3. Equivalent dead layer

The monolithic silicon telescope is equipped with a titanium-based dead layer of thickness \(a_{\text{Ti}}\) (0.24 \(\mu\)m) that affects the response of the detector at the lowest energies. In order to carry out an accurate, but simple, description of the response functions, the presence of this layer will be taken into account introducing an equivalent silicon dead layer of thickness \(a\). From this point of view, the thickness \(a\) can be estimated by:

\[
a = a_{\text{Ti}} \cdot \zeta_{\text{Si}}
\]  

(3)

where the equivalence factor \(\zeta_{\text{Si}}\) is the ratio of stopping power \(S^{\text{Si}}(E_p)\) of protons in silicon to that in titanium \(S^{\text{Ti}}(E_p)\) averaged over the energy range of interest. With a maximum proton energy of 8 MeV, \(\zeta_{\text{Si}}\) results to be 1.62. Thus, from eq. (3), the thickness of the equivalent silicon dead layer \(a\) results to be about 0.38 \(\mu\)m.

III.4. The radiator-detector interface distribution

The goal is to calculate the distributions of the energy deposited in the two stages of the silicon telescope, namely \(\Delta E\) and \(E\). The geometrical structure of the detection system and the interaction geometry of fast neutrons in the radiator are sketched in fig. 3. Of course, polyethylene and silicon have to be treated separately. Therefore, the detection system analysis will be subdivided into two different steps. Starting from the initial distribution of the recoil-protons, the calculation of their energy distribution \(p(E_p^{\text{int}}|\cos\theta)\) in polyethylene at different angles up to the interface radiator-detector will be firstly carried out. Then, using \(p(E_p^{\text{int}}|\cos\theta)\) like a “source distribution”, the distributions of energy deposited in the \(\Delta E\) and
E silicon stages, \( p(E_{d}^{\Delta E}) \) and \( p(E_{d}^{E}) \) respectively, will be evaluated. It should be pointed out that the effect of the radiator is to generate a proton field with the double differential distribution given by \( p(E_{p}^{\text{int}} | \cos \theta) \). Nothing would change if the radiator was substituted with an hypothetical proton source generating a field described by \( p(E_{p}^{\text{int}} | \cos \theta) \).

The recoil-proton energy at the radiator-detector interface \( E_{p}^{\text{int}} \) coincides with the energy \( E_{d} \) released to a one-stage detector of infinite thickness without dead-layer and directly coupled to the radiator. In this first step, the calculation of \( E_{p}^{\text{int}} \) does not depend on the detector material, which will be considered in polyethylene.

![Interaction geometry for neutron energy \( E_{n} \)](image)

The calculation of the recoil-proton spectrum at different angles can be performed by adopting a geometrical approach where the proton path is assumed to be straight. By observing the scheme shown in fig. 3, the range \( R_{\text{poly}}(E_{p}^{\text{int}}) \) of a recoil proton of energy \( E_{p}^{\text{int}} \) is given by the difference between its range \( R_{\text{poly}}(E_{n} \cdot \cos^{2} \theta) \) at the initial energy \( E_{p} = E_{n} \cdot \cos^{2} \theta \) and the distance traveled in the radiator at the emission direction \( \theta \), that is:

\[
R_{\text{poly}}(E_{p}^{\text{int}}) = R_{\text{poly}}(E_{n} \cdot \cos^{2} \theta) - \frac{x}{\cos \theta}
\]  

(4)
where \( x \) is the distance between the neutron interaction point (obviously corresponding to the generation point of the recoil-proton) and the radiator-detector interface. The interactions occurring at distances from the radiator-detector interface which are higher than the range \( R_{\text{poly}}(E_n) \) of a recoil-proton generated in a head-on collision can be neglected. In other words, for a given energy \( E_n \), \( x \leq R_{\text{poly}}(E_n) \). In fact, recoil-protons generated at higher distances cannot cross the interface and do not release any energy below the radiator. This consideration permits to identify a “radiator layer of interest” with a thickness equal to \( R_{\text{poly}}(E_n) \) and over which the evaluation of the probability of interaction of the impinging neutrons can be limited. Moreover, due to the low cross section of the neutron elastic scattering with hydrogen, the interaction point may be chosen uniformly along the radiator layer of interest.

By inverting this relation, it is easy to calculate \( E_{p,\text{int}} \) with the inverse function of \( R_{\text{poly}}(E_p) \), that will be defined as \( E_{\text{poly}}(R_p) \). For a given emission angle \( \theta \) (Fig. 3), the maximum value of \( E_{p,\text{int}} \) is \( E_n \cdot \cos^2 \theta \) and corresponds to the recoil energy of protons generated at \( x = 0 \) (i.e. at the radiator-detector interface), while the minimum one is 0.

The probability density \( p(E_{p,\text{int}} | \cos \theta) \) associated to an energy \( E_{p,\text{int}} \) at a definite angle \( \theta \) as:

\[
p(E_{p,\text{int}} | \cos \theta) \cdot d(E_{p,\text{int}} | \cos \theta) = p(x) \cdot dx
\]

or also

\[
p(E_{p,\text{int}} | \cos \theta) = p(x) \cdot \frac{dx}{d(E_{p,\text{int}} | \cos \theta)}
\]

From the previous considerations, the probability density distribution of \( x \) is uniform between 0 and \( R_{\text{poly}}(E_n) \), therefore:

\[
p(x) = \frac{1}{R_{\text{poly}}(E_n)}
\]

From eq. (4) \( x \) can be expressed as:

\[
x = \left[ R_{\text{poly}}(E_n \cdot \cos^2 \theta) - R_{\text{poly}}(E_{p,\text{int}}) \right] \cdot \cos \theta
\]
and differentiating equation (8):

\[
\frac{dx}{d(E_p^{\text{int}} | \cos \theta)} = -\frac{dR^{\text{poly}}(E_p^{\text{int}})}{d(E_p^{\text{int}} | \cos \theta)} \cdot \cos \theta
\]  \hspace{1cm} (9)

The derivative in the right-hand member of the eq. (9) is equal, by definition, to the inverse of the stopping power of protons in polyethylene at the energy \(E_p^{\text{int}}\), \(S^{\text{poly}}(E_p^{\text{int}})\). By substituting eq. (9) in eq. (6), and taking into account eq. (7), the probability density distribution of \(E_p^{\text{int}}\) at a definite angle \(\theta\) is:

\[
p(E_p^{\text{int}} | \cos \theta) = \left[R^{\text{poly}}(E_n)\right]^{-1} \cdot \frac{dR^{\text{poly}}(E_p^{\text{int}})}{d(E_p^{\text{int}} | \cos \theta)} \cdot \cos \theta = \left[R^{\text{poly}}(E_n) \cdot S^{\text{poly}}(E_p^{\text{int}})\right]^{-1} \cdot \cos \theta
\]  \hspace{1cm} (10)

Some profiles of the \(p(E_p^{\text{int}} | \cos \theta)\) distribution are shown in fig.4 for 3 MeV neutrons at different angles \(\theta\).

Fig. 4  Probability density function of the recoil-proton energy at radiator-detector interface for different emission angles \(\theta\). The distribution is normalized to a single neutron interaction and \(E_n\) is the incident neutron energy.
The probability that a recoil-proton emitted at an angle $\theta$ crosses the radiator-detector interface can be calculated by integrating eq. (10) over $E_p^{\text{int}}$ in the interval $(0 - E_n\cos^2 \theta)$:

$$p_{\text{int}}(\cos \theta) = \int_0^{E_n \cdot \cos^2 \theta} p(E_p^{\text{int}} \mid \cos \theta) \cdot dE_p^{\text{int}} = \frac{E_n \cdot \cos^2 \theta}{\left[R_{\text{poly}}(E_n)\right]^1} \cdot \frac{\frac{dR_{\text{poly}}(E_p^{\text{int}})}{d(E_p^{\text{int}} \mid \cos \theta)}}{\cos \theta \cdot dE_p^{\text{int}}}$$

which results to be:

$$p_{\text{int}}(\cos \theta) = \frac{R_{\text{poly}}(E_n \cdot \cos^2 \theta) \cdot \cos \theta}{R_{\text{poly}}(E_n)}$$  \hspace{1cm} (12)

Expression (12) can be directly obtained by observing that, at a definite angle $\theta$, the probability that a recoil-proton crosses the radiator-detector interface is given by the ratio of the thickness of the radiator “layer of interest” for protons emitted at $\theta$ (i.e. $R_{\text{poly}}(E_n \cdot \cos^2 \theta) \cdot \cos \theta$) to that of the maximum layer of interest, which is defined at $\theta = 0^\circ$ ($R_{\text{poly}}(E_n)$).

The total probability $p_{\text{int}}$ of crossing the radiator-detector interface can be calculated by integrating expression (12) over $\cos^2 \theta$, by exploiting the uniform distribution over $\cos^2 \theta$ of

![Fig. 5](image_url)  
**Fig. 5** Intrinsic efficiency of a detector directly coupled to the radiator at different neutron energies.
s-wave scattering. The intrinsic efficiency of the detection system at a given neutron energy can be obtained by multiplying $p_{\text{int}}$ by the probability of interaction $\pi(En)$ (eq. (2)). The intrinsic efficiency is plotted in fig. 5 for different neutron energies and a sensitive area of 1 mm$^2$. The curve was plotted by using the range data taken from ref. 34.

Above 300 keV, the intrinsic efficiency of a detector directly coupled to the radiator is of the order of $10^{-5}$ counts per unit neutron fluence, as confirmed by the experimental data discussed in chapter VI, section 3.

III.5. Response functions (total and E stage)

The probability density distribution $p(E_{p}^{\text{int}} | \cos \theta)$, given by eq. (10), is the “source distribution” for the calculation of the response functions of the two telescope stages. From this expression it is also possible to obtain the complete $p(E_{p}^{\text{int}})$ distribution by integrating the $\cos \theta$ function for an assigned $E_{p}^{\text{int}}$, $(\cos \theta|E_{p}^{\text{int}})_{\text{tot}}$, over $\cos^2 \theta$, by taking into account the uniform distribution over $\cos^2 \theta$ of the s-wave scattering.

The lower limit of integration for $\cos \theta$ can be derived by observing the behavior of $p(E_{p}^{\text{int}} | \cos \theta)$ (fig.3) at a fixed $E_{p}^{\text{int}}$ value. This lower limit corresponds to the maximum angle $\theta$ at which the energy $E_{p}^{\text{int}}$ can be deposited, that can be evaluated from the relation $E_{p}^{\text{int}} = E_n \cdot \cos^2 \theta$. On the other hand, the higher limit is simply $\cos \theta = 1$. Therefore:

\[
(cos \theta | E_{p}^{\text{int}})_{\text{tot}} = \int_{E_{p}^{\text{int}}}^{1} \cos \theta \cdot d \cos^2 \theta
\]

which results to be:

\[
(cos \theta | E_{p}^{\text{int}})_{\text{tot}} = \frac{2}{3} \left[ 1 - \left( \frac{E_{p}^{\text{int}}}{E_n} \right)^{\frac{3}{2}} \right]
\]

(14)

Taking into account exp. (10), the probability density distribution of $E_{p}^{\text{int}}$ is given by:

\[
p(E_{p}^{\text{int}}) = \frac{2}{3} \left[ 1 - \left( \frac{E_{p}^{\text{int}}}{E_n} \right)^{\frac{3}{2}} \right] \cdot \left[ R^{\text{poly}} (E_n) \cdot S^{\text{poly}} (E_{p}^{\text{int}}) \right]^{-1}
\]

(15)

Of course, this is also the probability density function of the energy deposited below the radiator per unit neutron interaction. Fig. 6 shows a comparison between the distribution
described by eq. (15) and the results from the reference [31] for several neutron energies. The discrepancies observed below 1 MeV are associated to the approximations adopted in ref. 31.

For a given angle $\theta$, eq. (10) gives the probability density $p(E_{p}^{\text{int}} \mid \cos \theta)$ that a recoil-proton of energy within the interval $(E_{p}^{\text{int}} \div E_{p}^{\text{int}+dE_{p}^{\text{int}}})$ crosses the radiator-detector interface per unit neutron interaction. This distribution describes the proton field at the detector surface and can be thought as the field produced by an imaginary “source” placed in contact with the detector. At this point, it is possible to investigate what happens in the silicon detector by ignoring the presence of the radiator.

Following the same procedure, it is possible to calculate the distribution of energy deposited beyond the dead layer, i.e. the distribution of the total energy $E_{d}^{\text{tot}}$ deposited in the sensitive volume of the entire telescope:

$$E_{d}^{\text{tot}} = E_{d}^{\Delta E} + E_{d}^{E}$$ (16).

Also in this case one should assume that the path of the recoil-protons is a pure straight line. Under this hypothesis, for an assigned emission angle $\theta$, the range in silicon $R_{Si}^{E_{d}^{\text{tot}}}$ of a recoil proton of energy $E_{d}^{\text{tot}}$ is given by the difference between its range at the energy $E_{p}^{\text{int}}$ (at

![Fig. 6](image-url) Fig. 6  Comparison between the $E_{p}^{\text{int}}$ distribution from this model (red line) and the expression given in ref. [31]
the detector surface) and the distance traveled in the dead layer of thickness a:

$$R_{Si}^{\text{tot}}(E_d^{\text{tot}}) = R_{Si}^{\text{int}}(E_p^{\text{int}}) - \frac{a}{\cos \theta} \quad (17)$$

As already mentioned, for a given angle $\theta$, the maximum value of $E_p^{\text{int}}$, $E_p^{\text{int}}_{\text{max}} = E_n \cos^2 \theta$. Thus, the maximum value of $E_d^{\text{tot}}$ at each angle $\theta$ can be obtained by substituting $E_p^{\text{int}}_{\text{max}}$ in eq. (17) and afterwards by inverting it:

$$E_d^{\text{tot}}_{\text{max}} = E_{Si}^{-1} \left[ R_{Si}^{\text{int}}(E_n \cdot \cos^2 \theta) - \frac{a}{\cos \theta} \right] \quad (18)$$

where the function $E_{Si}^{-1}$ is the inverse function of the proton range $R_{Si}^{\text{int}}(E_p)$ in silicon. It should be underlined that $E_{Si}^{-1}$ can be evaluated directly from the range-energy data available in the literature. Of course, $E_d^{\text{tot}}_{\text{max}} < E_n \cos^2 \theta$, because an amount of energy is always deposited in the dead layer.

Since $E_d^{\text{tot}}$ and $E_p^{\text{int}}$ are correlated, at a fixed $\theta$ angle their associated distribution functions are constrained by the equation:

$$p(E_d^{\text{tot}} | \cos \theta) \cdot d(E_d^{\text{tot}} | \cos \theta) = p(E_p^{\text{int}} | \cos \theta) \cdot d(E_p^{\text{int}} | \cos \theta) \quad (19)$$

where $p(E_d^{\text{tot}} | \cos \theta) > 0$.

In order to extract $p(E_d^{\text{tot}} | \cos \theta)$, it is necessary to deduce $E_p^{\text{int}}$ from eq. (17), that is:

$$E_p^{\text{int}} = E_{Si}^{-1} \left[ R_{Si}^{\text{int}}(E_d^{\text{tot}}) + \frac{a}{\cos \theta} \right] \quad (20)$$

$p(E_d^{\text{tot}} | \cos \theta)$ can be determined from eq.(19):

$$p(E_d^{\text{tot}} | \cos \theta) = p(E_p^{\text{int}} | \cos \theta) \cdot \frac{d(E_p^{\text{int}} | \cos \theta)}{d(E_d^{\text{tot}} | \cos \theta)} \quad (21)$$
The derivative at the right-hand of eq. (21) can be rearranged in the following way, by taking into account the eq (17):

\[
\frac{d(E_p^{\text{int}} | \cos \theta)}{d(E_d^{\text{tot}} | \cos \theta)} = \frac{d(E_p^{\text{int}} | \cos \theta)}{d(R_S^{\text{Si}} (E_p^{\text{int}}))} \cdot \frac{d(R_S^{\text{Si}} (E_p^{\text{int}}))}{d(E_d^{\text{tot}} | \cos \theta)} = \frac{d(E_p^{\text{int}} | \cos \theta)}{d(R_S^{\text{Si}} (E_d^{\text{tot}}))} \cdot \frac{d(R_S^{\text{Si}} (E_d^{\text{tot}}))}{d(E_d^{\text{tot}} | \cos \theta)} = \frac{S^{\text{Si}} (E_p^{\text{int}})}{S^{\text{Si}} (E_d^{\text{tot}})} \tag{22}
\]

Eqs. (10), (21) and (22) give:

\[
p(E_d^{\text{tot}} | \cos \theta) = \frac{1}{R^{\text{poly}} (E_n)} \cdot \frac{S^{\text{Si}} (E_p^{\text{int}})}{S^{\text{poly}} (E_p^{\text{int}})} \cdot \frac{1}{S^{\text{Si}} (E_d^{\text{tot}})} \cdot \cos \theta \tag{23}
\]

Finally, by exploiting eq. (20):

\[
p(E_d^{\text{tot}} | \cos \theta) = \frac{1}{R^{\text{poly}} (E_n)} \cdot \frac{S^{\text{Si}} (E_d^{\text{tot}})}{S^{\text{poly}} (E_d^{\text{tot}})} \cdot \frac{1}{S^{\text{Si}} (E_d^{\text{tot}})} \cdot \cos \theta \tag{24}
\]

This distribution is valid in the interval \((0 \div E_{d^{\text{tot}}}^{\text{max}})\).

In order to obtain the distribution of total deposited energy \(p(E_d^{\text{tot}})\), this expression should be integrated over all the possible \(\cos \theta\) values (by taking into account the uniform distribution over \(d \cos^2 \theta\) of s-wave scattering). The higher limit of integration is \(\cos \theta = 1\). The lower limit is the cosine of the maximum angle at which an amount of energy \(E_d^{\text{tot}}\) can be deposited. This value should be derived numerically from eq. (17) after the substitution \(E_p^{\text{int}} = E_n \cdot \cos^2 \theta\):

\[
R^{\text{Si}} (E_d^{\text{tot}}) = R^{\text{Si}} (E_n \cdot \cos^2 \theta_{\text{max}} | E_d^{\text{tot}}) - \frac{a}{\cos(\theta_{\text{max}} | E_d^{\text{tot}})} \tag{25}
\]

The probability density distribution of the total energy deposited in the telescope can be calculated numerically from the following integral:
\[ p(E_d^{\text{tot}}) = \int_{\cos^2 \theta = \cos^2 (\theta_{\text{max}}^{E_d^{\text{tot}}})}^{\cos^2 \theta = 1} p(E_d^{\text{tot}} | \cos \theta) \cdot d \cos^2 \theta \]

Alternatively, the calculation of \( p(E_d^{\text{tot}}) \) can be carried out analytically by exploiting the following approximation:

\[
\int_{\cos^2 (\theta_{\text{max}}^{E_d^{\text{tot}}})}^{1} \frac{S_{\text{Si}}^E \left( E_{\text{Si}} \left( R_{\text{Si}}^{E_d^{\text{tot}}} + \frac{a}{\cos \theta} \right) \right)}{S_{\text{poly}}^E \left( E_{\text{Si}} \left( R_{\text{Si}}^{E_d^{\text{tot}}} + \frac{a}{\cos \theta} \right) \right)} \cdot \cos \theta \cdot d \cos^2 \theta \cong \int_{\cos^2 (\theta_{\text{max}}^{E_d^{\text{tot}}})}^{1} \frac{S_{\text{Si}}^E \left( E_{\text{Si}} \left( R_{\text{Si}}^{E_d^{\text{tot}}} + \frac{a}{\cos \theta} \right) \right)}{S_{\text{poly}}^E \left( E_{\text{Si}} \left( R_{\text{Si}}^{E_d^{\text{tot}}} + \frac{a}{\cos \theta} \right) \right)} \cdot \cos \theta \cdot d \cos^2 \theta
\]

The trend of the relative error of the approximation made above (eq. (27)) is shown in fig. 7 for \( E_n = 8 \) MeV. It should be underlined that this approximation is lower than 1% at energies higher than 50 keV. Thus, the ratio of the stopping powers can be brought outside the integral, and eq. (26) can be reduced to the simple form:
\[ p(E_d^{\text{tot}}) = \frac{2}{3} \left[ 1 - \cos(\theta_{\text{max}}) \left| E_d^{\text{tot}} \right|^3 \right] \frac{1}{R^\text{poly}(E_n)} \cdot \frac{S^{\text{Si}}(E_p^{\text{int}})}{S^{\text{poly}}(E_p^{\text{int}})} \cdot \frac{1}{S^{\text{Si}}(E_d^{\text{tot}})} \]

This distribution represents the probability density of depositing an amount of energy within the interval \((E_d^{\text{tot}} + E_d^{\text{tot}} + dE_d^{\text{tot}})\) per unit neutron interaction in the total sensitive volume of the telescope, i.e. the sum of the \(\Delta E\) and the E stage.

It should be pointed out that the response function of the E stage is based on the same structure of expression (28). In fact, the two cases differ only for the thickness of the material placed between the radiator and the detector. It is sufficient to substitute the quantity \(a\) in eqs. (25) and (28) with the sum of \(a\) plus the thickness \(h\) of \(\Delta E\) stage, that is:

\[ p(E_d^E \mid \cos \theta) = \frac{1}{R^\text{poly}(E_n)} \cdot \frac{S^{\text{Si}}(E_p^{\text{int}})}{S^{\text{poly}}(E_p^{\text{int}})} \cdot \frac{1}{S^{\text{Si}}(E_d^E)} \cdot \cos \theta \]

\[ R^{\text{Si}}(E_d^E) = R^{\text{Si}}(E_n \cdot \cos^2(\theta_{\text{max}}) \mid E_d^E)) - \frac{a + h}{\cos(\theta_{\text{max}}) \mid E_d^E} \]

\[ p(E_d^E) = \frac{2}{3} \left[ 1 - \cos(\theta_{\text{max}}) \left| E_d^E \right|^3 \right] \frac{1}{R^\text{poly}(E_n)} \cdot \frac{S^{\text{Si}}(E_p^{\text{int}})}{S^{\text{poly}}(E_p^{\text{int}})} \cdot \frac{1}{S^{\text{Si}}(E_d^E)} \]

where \(0 \leq E_d^E \leq E_d^E_{\text{max}}\), with

\[ E_d^E_{\text{max}} = E^{\text{Si}} \left[ R^{\text{Si}}(E_n \cdot \cos^2 \theta) - \frac{a + h}{\cos \theta} \right] \]

The probability density distribution \(p(E_d^E)\) (eq. (31)) is normalized to a single neutron interaction. For a quantitative evaluation, it must be multiplied by the corresponding probability \(\pi(E_n)\) of generating a recoil-proton in the radiator per unit neutron fluence expressed by eq. (2).

In order to validate this step of the analytical procedure, the response function of the E stage was calculated with Monte Carlo simulations by using the FLUKA code [36-39]. The simulation geometry consisted of a polyethylene converter 1 mm thick placed in contact with a silicon telescope with a 0.25 µm thick titanium dead layer, a \(\Delta E\) stage (thickness 1.9 µm)
and an E stage (thickness 500 µm). The source was a parallel beam of mono-energetic neutrons. The comparison between the analytical and simulation results is shown in fig 8. The agreement is very satisfactory and demonstrates the validity of the procedure described above. It should be underlined that the Monte Carlo simulations take into account the particle straggling. Despite of the simplifications adopted, the analytical model is able to reproduce properly the average behavior of response functions at all the energies considered. The intrinsic efficiency of the E stage expressed in counts per unit neutron fluence can be determined by integrating eq. (31) from 0 to E_{d,E}^{\text{max}}, and multiplying the result by \pi(E_n) (eq. (2)).

![Response functions of E stage at different neutron energies calculated analytically (black line) and with Monte Carlo simulations (by using the FLUKA code) (magenta histogram).](image)

**Fig. 8** Response functions of E stage at different neutron energies calculated analytically (black line) and with Monte Carlo simulations (by using the FLUKA code) (magenta histogram).

### III.6. ΔE stage analysis

According to the energy balance expressed by eq. (16), the response function of the ΔE stage can be calculated in a simple way by assuming that, for a definite θ angle, the energy deposited in a single event by a proton with energy E_{p,int} at the radiator-detector interface is equal to the difference between the energy E_{d,tot} deposited in the telescope as a whole and the energy E_{d,E} deposited in the E stage. The distribution functions of E_{p,int} and E_{d,\Delta E} are connected.
by an equation similar to eq. (19). Nevertheless, the analysis is complicated by some mathematical difficulties (which reflect the physical situation, as it will be underlined in the following).

Starting from eq. (17), the following expression can be obtained:

\[
E_d^{AE} = E_d^{tot} - E_d^E = E^{Si}\left( R^{Si}(E_p^{int}) - \frac{a}{\cos \theta} \right) - E^{Si}\left( R^{Si}(E_p^{int}) - \frac{a + h}{\cos \theta} \right)
\]  (33)

for any \( E_p^{int} \) in the interval \((0 \div E_a \cos^2 \theta)\). This is the mathematical expression of \( E_d^{AE} \) as a function of \( E_p^{int} \) at different \( \theta \) angles. Three different intervals can be identified (it should be remembered that \( E^{Si}(R) \) is the inverse function of the proton range in silicon):

i) \( E_p^{int} \leq E^{Si}\left( \frac{a}{\cos \theta} \right) \), where both the \( E_d^{tot} \) and \( E_d^E \) are undefined because the arguments of the function \( E^{Si} \) are negative; this correspond to the physical situation in which the proton stops inside the dead layer and does not release energy in the detector.

ii) \( E^{Si}\left( \frac{a}{\cos \theta} \right) < E_p^{int} \leq E^{Si}\left( \frac{a + h}{\cos \theta} \right) \), where \( E_d^{tot} \geq 0 \) while \( E_d^E \) is still undefined; in this case the proton emitted at the \( \theta \) angle reaches the \( \Delta E \) stage and stops inside it. For this reasons, this class of protons are called stoppers (a terminology peculiar to microdosimetry). In particular, those protons whose energy is exactly equal to the higher limit \( E^{Si}\left( \frac{a + h}{\cos \theta} \right) \), or the protons that stop exactly at the \( \Delta E \)-E stage interface, are called exact stoppers.

iii) \( E^{Si}\left( \frac{a + h}{\cos \theta} \right) < E_p^{int} \), where both the \( E_d^{tot} \) and \( E_d^E \) are positive; here the proton cross the \( \Delta E \) stage and stops inside the E stage. These protons are called crossers.
Fig. 9 Energy deposited in the $\Delta E$ stage $E_d^{\Delta E}$ by recoil-protons of energy $E_p^{\text{int}}$, at different scattering angles $\theta$. Stoppers and crossers are plotted with different colors.

Fig. 9 shows some curves obtained from eq. (33) for $E_n = 2.727$ MeV, at four different angles $\theta$. In this figure the different classes of protons are plotted with different colors. It should be noted that the maximum energy deposited in the $\Delta E$ stage, $E_d^{\Delta E \text{max}}$, is released by crossers having an energy slightly higher than that of the exact stoppers. These crossers are the protons whose stopping powers assume the same values when passing from the entrance to the exit edges of the $\Delta E$ stage. In fact, the derivate of eq. (33) gives:

$$
\frac{dE_d^{\Delta E}}{dE_p^{\text{int}}} = \frac{d}{dE_p^{\text{int}}} E^{\text{Si}}\left( R^{\text{Si}}(E_p^{\text{int}}) - \frac{a}{\cos \theta} \right) - \frac{d}{dE_p^{\text{int}}} E^{\text{Si}}\left( R^{\text{Si}}(E_p^{\text{int}}) - \frac{a + h}{\cos \theta} \right)
$$

(34)

By elaborating the second member, the following expression can be obtained:

$$
\frac{dE_d^{\Delta E}}{dE_p^{\text{int}}} = \frac{S^{\text{Si}}\left( E^{\text{Si}}\left( R^{\text{Si}}(E_p^{\text{int}}) - \frac{a}{\cos \theta} \right) \right)}{S^{\text{Si}}(E_p^{\text{int}})} - \frac{S^{\text{Si}}\left( E^{\text{Si}}\left( R^{\text{Si}}(E_p^{\text{int}}) - \frac{a + h}{\cos \theta} \right) \right)}{S^{\text{Si}}(E_p^{\text{int}})}
$$

(35)
The maximum value of \( E_{d,\Delta E} \) corresponds to the recoil-proton energy \( E_{p,\text{int}} \) at which the derivative expressed by (35) is equal to zero, thus to the \( E_{p,\text{int}} \) value that satisfies the equation:

\[
S^\text{Si} \left( E^\text{Si} \left( R^\text{Si} (E_{p,\text{int}}) - \frac{a}{\cos \theta} \right) \right) = S^\text{Si} \left( E^\text{Si} \left( R^\text{Si} (E_{p,\text{int}}) - \frac{a + h}{\cos \theta} \right) \right)
\]

or, by exploiting eq. (17):

\[
S^\text{Si} (E_{d,\text{tot}}) = S^\text{Si} (E_{d,\text{E}})
\]

Since \( E_{d,\text{tot}} \) and \( E_{d,\text{E}} \) can be regarded as the recoil-proton energy at the entrance and at the exit of the \( \Delta E \) stage, respectively, \( E_{d,\Delta E,\text{max}} \) is released by those recoil-protons whose energy trend satisfies equation (37).

In order to simply the mathematical procedure and to better understand the behavior of the \( \Delta E \) stage of the telescope, the calculation will be performed by discriminating the classes of protons, i.e. stoppers and crossers.

**Stoppers**

For a stopper, i.e. a particle which stops inside the \( \Delta E \) stage, the \( \Delta E \) layer behaves like detector with an “infinite” thickness. Thus their energy deposition at different emission angles \( \theta \) can be described by the same distribution of \( E_{d,\text{tot}} \), given by eq. (24), by taking into account the proper definition limits:

\[
p(E_{d,\Delta E,\text{s}} | \cos \theta) = \frac{1}{R_{\text{poly}} (E_n)} \cdot S^\text{Si} \left( E^\text{Si} \left( R^\text{Si} (E_{d,\Delta E,\text{s}}) + \frac{a}{\cos \theta} \right) \right) \cdot \frac{1}{S^\text{Si} (E_{d,\Delta E,\text{s}})} \cdot \cos \theta
\]

where \( E_{d,\Delta E,\text{s}} \) represents the energy deposited in the \( \Delta E \) stage by stoppers and is defined from 0 to the minimum between these two values:

1) \( E^\text{Si} \left( \frac{h}{\cos \theta} \right) \), the maximum energy deposited by stoppers at definite \( \theta \) angle when, according to the interaction geometry, protons can reach the \( \Delta E-E \) stages interface. At the angle \( \theta \), the incident neutron energy is such that \( R^\text{Si} (E_n \cos^2 \theta) > \frac{a + h}{\cos \theta} \) and it is necessary to exclude the contribution of crossers from the distribution given by eq (24);
2) \( E_{dS} = \left( R_{Si} = (E_n \cos^2 \theta) - \frac{a}{\cos \theta} \right) \), the maximum \( E_{d}^{\Delta E} \) value when only stoppers are generated at the \( \theta \) angle.

Fig. 10 shows some distributions obtained at different angles from eq. (24) and (38). It should be observed that, at the definite neutron energy, by increasing \( \theta \), the maximum energy deposited by the stoppers shifts towards higher values up to a limiting value of \( \theta \). Above this value it decreases. This limiting angle \( \Theta_{lim} \) represents the angle above which no recoil-protons cross the \( \Delta E \) stage. \( \Theta_{lim} \) can be obtained by solving the equation

\[
R_{Si} = \frac{a + h}{\cos \theta}.
\]

Now, the distribution of the energy deposited in the \( \Delta E \) stage by the stoppers \( E_{d,s}^{\Delta E} \) can be obtained by integrating eq. (38) over \( \cos^2 \theta \). The limits of integration can be determined with the arguments discussed below. By considering the analogy with the calculation of \( E_{d}^{\text{tot}} \) distribution and following the integrations paths shown in fig. 10, it can be deduced that the lower integration limit of \( \cos \theta \) is always \( \cos(\theta_{\text{max}}|E_{d,s}^{\Delta E}) \), while the higher one is \( \frac{h}{R_{Si}(Ed)} \) for \( E_{d,s}^{\Delta E} \) included in the interval from \( E_{d,s}^{\Delta E} \) to \( E_{d,s}^{\Delta E} \max \).

\[
\int \cos^2 \theta \, d\cos \theta = \frac{h}{R_{Si}(Ed)}.
\]

**Fig. 10** Stopper distributions \( p(E_{d,s}^{\Delta E} | \cos \theta) \) normalized to a single neutron interaction at different scattering angles \( \theta \). The integration paths for the calculation of \( p(E_{d,s}^{\Delta E}) \) are also plotted for three \( E_{d,s}^{\Delta E} \) values (black vertical lines).
Thus:

\[
p(E_{d,s}^{\Delta E}) = \frac{2}{3} \left[ 1 - \cos(\theta_{\text{max}} | E_{d,s}^{\Delta E}) \right]^3 \cdot \frac{1}{R^\text{poly}_n} \cdot \frac{S^\text{Si} \left( E^\text{Si} \left( R^\text{Si}_n (E_{d,s}^{\Delta E}) + a \right) \right)}{S^\text{poly} \left( E^\text{Si} \left( R^\text{Si}_n (E_{d,s}^{\Delta E}) + a \right) \right)} \cdot \frac{1}{S^\text{Si} (E_{d,s}^{\Delta E})}
\]

for \( 0 \leq E_{d,s}^{\Delta E} \leq E^\text{Si} (h) \), and

\[
p(E_{d,s}^{\Delta E}) = \frac{2}{3} \left[ \left( \frac{h}{R^\text{Si}_n (E_{d,s}^{\Delta E})} \right)^3 - \cos(\theta_{\text{max}} | E_{d,s}^{\Delta E}) \right] \cdot \frac{1}{R^\text{poly}_n} \cdot \frac{S^\text{Si} \left( E^\text{Si} \left( R^\text{Si}_n (E_{d,s}^{\Delta E}) + a \right) \right)}{S^\text{poly} \left( E^\text{Si} \left( R^\text{Si}_n (E_{d,s}^{\Delta E}) + a \right) \right)} \cdot \frac{1}{S^\text{Si} (E_{d,s}^{\Delta E})}
\]

for \( E^\text{Si} (h) < E_{d,s}^{\Delta E} \leq E^\text{Si} \left( \frac{h}{\cos \Theta_{\text{lim}}} \right) \) \hspace{1cm} (39)

The results of the numerical integration expressed by eq. (39) at different neutron energies are shown in fig. 11. These distributions are compared with the results of Monte Carlo simulations performed with the FLUKA code in the same figure. The agreement is good and it should be concluded that the model can reproduce the stopper distribution at all the neutron energies considered.

![Graph showing the distribution of stoppers in ΔE stage at different neutron energies calculated analytically and with Monte Carlo simulations.](Figure11)
Crossers

Following the same procedure described above, it is possible to calculate the distribution of energy $E_{d,c}^\Delta \varepsilon$ deposited in the $\Delta \varepsilon$ stage due to the crosser-type protons only. At a fixed $\theta$ angle, the distribution functions of $E_{p}^\text{int}$ and $E_{d}^\Delta \varepsilon$ are linked by the equation:

$$p(E_{d,c}^\Delta \varepsilon | \cos \theta) \cdot d(E_{d,c}^\Delta \varepsilon | \cos \theta) = p(E_{p}^\text{int} | \cos \theta) \cdot d(E_{p}^\text{int} | \cos \theta)$$  \hspace{1cm} (40)

The probability density $p(E_{d}^\Delta \varepsilon | \cos \theta)$ can be obtained from eq. (40) by considering that:

$$\frac{d(E_{p}^\text{int} | \cos \theta)}{d(E_{d,c}^\Delta \varepsilon | \cos \theta)} = \left[ \frac{d(E_{d,c}^\Delta \varepsilon | \cos \theta)}{d(E_{p}^\text{int} | \cos \theta)} \right]^{-1} = \left[ \frac{d(E_{d,c}^\Delta \varepsilon | \cos \theta)}{d(E_{d,c}^\text{int} | \cos \theta)} \right]^{-1} \cdot \left[ \frac{d(E_{d,c}^\text{int} | \cos \theta)}{d(E_{p}^\text{int} | \cos \theta)} \right]^{-1} = \left[ \frac{S_{\text{Si}}(E_{d,c}^\text{int})}{S_{\text{Si}}(E_{p}^\text{int})} \right]^{-1} \cdot \left[ \frac{S_{\text{Si}}(E_{d,c}^\text{int})}{S_{\text{Si}}(E_{p}^\text{int})} \right]^{-1}$$ \hspace{1cm} (41)

Substituting to $E_{d,c}^\text{int}$ and $E_{p}^\text{int}$ their respective inverse functions:

$$\frac{d(E_{p}^\text{int} | \cos \theta)}{d(E_{d,c}^\Delta \varepsilon | \cos \theta)} = S_{\text{Si}}(E_{p}^\text{int}) \cdot \left[ S_{\text{Si}} \left( E_{p}^\text{int} \left( R_{p}^\text{int} - \frac{a}{\cos \theta} \right) \right) \right]^{-1} \cdot \left[ S_{\text{Si}} \left( E_{p}^\text{int} \left( R_{p}^\text{int} - \frac{a + h}{\cos \theta} \right) \right) \right]^{-1}$$ \hspace{1cm} (42)

and, from eq. (31) and (10):

$$p(E_{d,c}^\Delta \varepsilon | \cos \theta) = \frac{\cos \theta}{R_{p}^\text{poly}(E_{p})} \cdot \frac{S_{\text{Si}}(E_{p}^\text{int})}{S_{\text{poly}}(E_{p}^\text{int})} \cdot \left[ S_{\text{Si}} \left( E_{p}^\text{int} \left( R_{p}^\text{int} - \frac{a}{\cos \theta} \right) \right) \right]^{-1} \cdot \left[ S_{\text{Si}} \left( E_{p}^\text{int} \left( R_{p}^\text{int} - \frac{a + h}{\cos \theta} \right) \right) \right]^{-1}$$ \hspace{1cm} (43)

where $E_{d,c}^\Delta \varepsilon$ and $E_{p}^\text{int}$ are related by the eq. (33). This expression, unfortunately equivocal, must be inverted numerically in order to obtain $E_{p}^\text{int}$ as a function of $E_{d,c}^\Delta \varepsilon$. Due to the complexity of this mathematical procedure, the evaluation of $p(E_{d,c}^\Delta \varepsilon)$ is not discussed here. A more comprehensive method will be described in the next section.

In order to calculate the total probability density of the energy deposited in the $\Delta \varepsilon$ stage, the contributions of the stoppers and crossers at each $E_{d,c}^\Delta \varepsilon$ value must be added.
III.7. $\Delta E$ – E scatter plot and bivariate distribution

The monolithic silicon telescope consists of two stages that can be modeled as two adjacent, but independent, silicon detectors with different thickness (of about two order of magnitude). Each of them can provide separate information about the characteristics of the secondary radiation field. The $\Delta E$ stage permits to estimate a quantity related to the rate of energy lost by the traversing particle, i.e. the stopping power, while the E stage can be used to measure the energy of the impinging charged particle. Of course, the two detectors could be used independently by measuring the spectra of energy deposited in each stage by means of two multi-channel analyzers, regardless the link between the acquired distributions. For simplicity, in the following, the spectrum of energy deposited in the $\Delta E$ stage and in the E stage will be called $\Delta E$-spectrum and E-spectrum, respectively.

Nevertheless, the peculiarity of this detection system is represented by the possibility of measuring the energy release in the two detectors and, at the same time, the correlation between these amount of energies for each penetrating particle. This can be done by acquiring the time-correlated signals generated in the two stages of the telescope event-by-event using a two-channel ADC in coincidence mode, i.e. the so-called $\Delta E$-E scatter plot in a two-dimensional space. Moreover, from the frequency distribution of the $\Delta E$-E pairs, it is possible to obtain the bivariate distribution of the energy deposited by the secondary charged particles in the two stages. It should be underlined that this bivariate distribution includes all the information about the spectra measured by the two stages. In fact, its marginal distribution with respect to the energy releases in the E stage gives the $\Delta E$-spectrum, while its marginal distribution with respect to the energy release in the $\Delta E$ stage gives the E-spectrum.

Therefore, a complete analytical description of the $\Delta E$-E scatter plot and of the $\Delta E$-E bivariate distribution can be useful to better understand the correlation between the information provided by the two stages, in other words the behavior of telescope as a whole.

From the analytical procedure described above, it is fairly easy to derive the relation between the energies deposited in the $\Delta E$ stage and in the E stage per event. Considering that, from eq (16):

$$E_d^{\Delta E} = E_d^{\text{tot}} - E_d^E$$

(44)
and, at a definite emission angle $\theta$:

$$R^{Si}(E^\text{tot}_d) = R^{Si}(E^E_d) + \frac{h}{\cos \theta} \quad (45)$$

By inverting eq. (45), and by substituting in eq (44), $E^\Delta E_d$ results to be:

$$E^\Delta E_d = E^{Si}(R^{Si}(E^E_d) + \frac{h}{\cos \theta}) - E^E_d \quad (46)$$

where $0 \leq E^E_d \leq E^E_d \text{ max.}$

Fig. 12 shows eq. (46) at different $\theta$ angles and at a neutron energy equal to 2.727 MeV. In order to better define the portion of space over which eq. (46) is defined, the
maximum value of $E^{\Delta E}$ at each $E_d^E$ value can be calculated by substituting its value $\cos (\theta_{\text{max}} \mid E_d^E)$ to $\cos \theta$ in eq. (46):

$$E_d^{\Delta E \text{ max}} = E_S^S \left( R_S^S (E_d^E) + \frac{h}{\cos(\theta \mid E_d^E)} \right) - E_d^E$$

(47)

with $0 \leq E_d^E \leq E_d^{\Delta E \text{ max}}$. On the other hand, the minimum value of $E^{\Delta E}$, $E_d^{\Delta E \text{ min}}$, at each $E_d^E$ value corresponds to the events forward directed and it is simply given by eq. (46) with $\cos \theta = 1$. These two limiting curves are plotted in fig. 12.

The evaluation of the bivariate probability density $p(E_d^{\Delta E}, E_d^E)$ of finding $E_d^{\Delta E}$ in the interval $(E_d^{\Delta E} \div E_d^{\Delta E} + dE_d^{\Delta E})$ and $E_d^E$ in the interval $(E_d^E \div E_d^E + dE_d^E)$ can be carried out by observing that, from eq. (46):

$$\cos \theta = \frac{h}{R_S^S (E_d^{\Delta E} + E_d^E) - R_S^S (E_d^E)}$$

(48)

Fixing $E_d^E$, a biunique relation links $E_d^{\Delta E}$ to $\cos \theta$. Moreover, according to the uniform distribution of $\cos^2 \theta$ from 0 to 1, the joint probability density $p(E_d^E, \cos \theta)$ is given by the following equation:

$$p(E_d^E, \cos \theta) = \frac{1}{R_{\text{poly}}(E_d^E)} \frac{S_S^S \left( E_S^S \left( R_S^S (E_d^E) + \frac{a + h}{\cos \theta} \right) \right)}{S_{\text{poly}} \left( E_S^S \left( R_S^S (E_d^E) + \frac{a + h}{\cos \theta} \right) \right)} \cdot \frac{1}{S_S^S (E_d^E)} \cdot \cos \theta$$

(49)

Thus, the bivariate probability distribution $p(E_d^{\Delta E}, E_d^E)$ can be obtained by the relation:

$$p(E_d^E, \cos \theta) \cdot dE_d^E \cdot d \cos^2 \theta = p(E_d^{\Delta E}, E_d^E) \cdot dE_d^{\Delta E} \cdot dE_d^E$$

(50)

and therefore:

$$p(E_d^{\Delta E}, E_d^E) = p(E_d^E, \cos \theta) \cdot \frac{d \cos^2 \theta}{dE_d^{\Delta E}}$$

(51)

Now:
\[
\frac{d\cos^2 \theta}{dE_d^{\Delta E}} = \frac{d}{dE_d^{\Delta E}} \left( \frac{h}{R_{Si}(E_d^{\Delta E} + E_d^{E}) - R_{Si}(E_d^{E})} \right)^2 = 2 \cdot \left( \frac{h^2}{(R_{Si}(E_d^{\Delta E} + E_d^{E}) - R_{Si}(E_d^{E}))^3} \right) \frac{d(R_{Si}(E_d^{\Delta E} + E_d^{E}))}{dE_d^{\Delta E}}
\]

and therefore:

\[
\frac{d\cos^2 \theta}{dE_d^{\Delta E}} = 2 \cdot \left( \frac{h^2}{(R_{Si}(E_d^{\Delta E} + E_d^{E}) - R_{Si}(E_d^{E}))^3} \right) 
S_{Si}(E_d^{\Delta E} + E_d^{E})
\]

Finally, from eqs. (51), (49) and (53):

\[
p(E_d^{\Delta E}, E_d^{E}) = \frac{1}{R_{poly}(E_n)} \cdot \frac{S_{Si}\left( E_{Si}\left( R_{Si}(E_d^{\Delta E}) + \frac{a+h}{h} \cdot \left( R_{Si}(E_d^{\Delta E} + E_d^{E}) - R_{Si}(E_d^{E}) \right) \right) \right)}{S_{poly}\left( E_{Si}\left( R_{Si}(E_d^{\Delta E}) + \frac{a+h}{h} \cdot \left( R_{Si}(E_d^{\Delta E} + E_d^{E}) - R_{Si}(E_d^{E}) \right) \right) \right)}, \quad \frac{1}{S_{Si}(E_d^{E})} 
\]

\[
\ldots 2 \cdot \left( \frac{h^3}{(R_{Si}(E_d^{\Delta E} + E_d^{E}) - R_{Si}(E_d^{E}))^4} \right) \cdot S_{Si}(E_d^{\Delta E} + E_d^{E})
\]

(54)

---

**Fig. 13** Bivariate distribution of energy deposited in \( \Delta E \) and \( E \) stages \( p(E_d^{\Delta E}, E_d^{E}) \) calculated analytically from eq. (54). The distribution is calculated for 2.727 MeV neutron energy and is normalized to unit neutron fluence.
with \(0 \leq E_d^E \leq E_d^E \max\), and \(E_d^{\Delta E \min} \leq E_d^{\Delta E} \leq E_d^{\Delta E \max}\).

At a given neutron energy \(E_n\), expression (54) represents the joint probability density of depositing a recoil-proton energy within the interval \((E_d^{\Delta E} \div E_d^{\Delta E} + dE_d^{\Delta E})\) in the \(\Delta E\) stage and a recoil-proton energy within the interval \((E_d^E \div E_d^E + dE_d^E)\) in the \(E\) stage, per unit neutron interaction. As an example, the plot of \(p(E_d^{\Delta E}, E_d^E)\) at \(E_n = 2.272\) MeV is shown in fig. 13.

The same \(\Delta E\)-\(E\) bivariate distribution was calculated with the FLUKA code. The energies deposited in the two stages of the telescope by mono-energetic neutrons were evaluated in each simulation in order to reproduce the experimental acquisition mode. The results are shown in fig. 14, together with those obtained from the analytical model. It should be remembered that, differently from the simulation, the analytical approach does not take into account the charged particle straggling. Anyway, the agreement is good at all the energies.

![Fig. 14 Bivariate distribution of energy deposited in \(\Delta E\) and \(E\) stages calculated by using Monte Carlo simulations (FLUKA code). The distribution is calculated for 2.727 MeV neutron energy and is normalized to unit neutron fluence. The boundaries of the analytical scatter plot shown in fig. 12 are also plotted.](image-url)
considered.

As already mentioned, from this distribution it is possible to derive all the information about the spectra measured by the telescope. In fact, the distributions \( p(E_d^{\Delta E}) \) and \( p(E_d^E) \) can be obtained by integrating numerically eq. (54) over \( E_d^E \) and \( E_d^{\Delta E} \), respectively, using the proper integration limits:

\[
p(E_d^{\Delta E}) = \int_{E_d^{\Delta E \min}}^{E_d^{\Delta E \max}} p(E_d^{\Delta E}, E_d^E) \cdot dE_d^E
\]

and

\[
p(E_d^E) = \int_{E_d^{\Delta E \min}}^{E_d^{\Delta E \max}} p(E_d^{\Delta E}, E_d^E) \cdot dE_d^{\Delta E}
\]

Obviously, it can be demonstrated that the curves given by eq. (55) at different neutron energies assume the same values of those given by eq. (31) for each \( E_d^E \) (fig. 6). Fig. 15 shows the response function of \( \Delta E \) stage crossers obtained by exploiting eq. (56) and the correspondent results of the FLUKA simulation.

---

**Fig. 15** Distribution of crossers in \( \Delta E \) stage at different neutron energies calculated analytically (black line) and with Monte Carlo simulations (by using the FLUKA code) (magenta histogram).
Finally, the comparison between the total (stoppers + crossers) $\Delta E$ stage response function calculated analytically and the results of FLUKA simulation is shown in fig. 16. The good agreement between the curves demonstrates the validity of the analytical procedure described above, and this is an important starting point for the analytical processing of the microdosimetric spectra measured with the $\Delta E$-detector. As it will be shown in the next chapter, this validation permits to optimize the analytical corrections of these spectra as well as to derive the necessary equivalence criteria.

![Fig. 16](image_url)
Chapter IV

Track Length Distributions and Shape Analysis

The main difference between a silicon microdosimeter and a TEPC consists in the shape of the sensitive volume. The monolithic telescope here discussed is characterized by a \( \Delta E \) stage with a rectangular parallelepiped sensitive volume, while those of the reference TEPCs are cylindrical or spherical. When the two detection systems are exposed to a radiation field, the secondary particles that intersect the sensitive volumes give rise to different track length distributions and, consequently, different profiles of the imparted energy spectra. The objective of this study is to individuate the optimum criteria to adopt in the inter-comparison between the solid state microdosimeter and a cylindrical TEPC. Thus, the role of the geometries in the microdosimetric measurements should be accurately investigated in order to individuate the optimum correction function for shape-equivalence.

A thorough investigation of the inter-comparison of microdosimetric measurements will be presented in chapter VII. The following sections of the present chapter, describe a theoretical model for the track length distribution in various sensitive volumes, basing on concepts of geometrical probability. It should be underlined that the general treatment was developed for spherical sensitive volumes and therefore it cannot be applied to the case of the silicon microdosimeter, because of its different detection geometry. An analytical procedure was developed in the framework of this thesis in order to describe the actual track length distribution of recoil-protons generated via elastic scattering in a polyethylene converter coupled to the monolithic silicon telescope.

A comparison with the track length distributions, based on criteria for the equivalence of volume shapes outlined by Kellerer [40] and, subsequently, by Bradley [21], will be finally discussed.
IV.1. Chord length distributions

The chord length distribution $c(l)$ represents the probability that a convex body of shape $\Sigma$, traversed by random straight lines of infinite length, intercepts a track of length $l$. In general, several types of randomness result from this intersection, depending on the constraints imposed to the cross of the straight lines in the interior of the body [41,42]. However, for the common microdosimetric experimental situations, the comparison between different shapes is typically based on the so-called $\mu$-randomness, or isotropic uniform randomness, where the body is exposed to a uniform isotropic field of infinite straight lines.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Chord Length distribution, $c(l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Sphere</strong></td>
<td>$\frac{2 \cdot l}{d^2}$ for $0 &lt; l &lt; d$ (d: diameter)</td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
<td>$2k \cdot \frac{1}{a} \left( \frac{h \cdot x^3}{\sqrt{1 - x^2}} \right) \cdot f(l \cdot x) - x^3 \cdot f(l \cdot x) + 2 \cdot x^2 \cdot F(l \cdot x) dx \right} + 2 \cdot \frac{1}{l^3} \cdot \int F(x) dx$</td>
</tr>
<tr>
<td></td>
<td>where $a = \sqrt{1 - h^2 / l^2}$ for $l &gt; h$, and the 2nd integral applies only to values $l \leq h$</td>
</tr>
<tr>
<td></td>
<td>$k = \frac{1}{\pi \cdot \left( \frac{d}{4} + \frac{h}{2} \right)}$, $f(t) = \frac{t}{d \cdot \sqrt{d^2 - t^2}}$ and $F(t) = \sqrt{1 - \frac{t^2}{d^2}}$</td>
</tr>
<tr>
<td></td>
<td>(d: diameter of the circular cross section, h: height)</td>
</tr>
<tr>
<td><strong>Cube</strong> [41]</td>
<td>$2k \cdot \frac{1}{a} \left( \frac{h \cdot x^3}{\sqrt{1 - x^2}} \right) \cdot f(l \cdot x) - x^3 \cdot f(l \cdot x) + 2 \cdot x^2 \cdot F(l \cdot x) dx \right} + 2 \cdot \frac{1}{l^3} \cdot \int F(x) dx$</td>
</tr>
<tr>
<td></td>
<td>where $a = \sqrt{1 - h^2 / l^2}$ for $l &gt; h$, and the 2nd integral applies only to values $l \leq h$</td>
</tr>
<tr>
<td></td>
<td>$k = \frac{1}{\pi \cdot \left( \frac{c}{4} + \frac{h}{2} \right)}$, $F(t) = \left{ \frac{1}{2c} \cdot t \cdot \frac{1}{2c - \sqrt{1 - c^2 / t^2}} \right}$ and $f(t) = -\frac{dF(t)}{dt}$</td>
</tr>
<tr>
<td></td>
<td>(c: side length, h: height)</td>
</tr>
<tr>
<td><strong>Infinite Slab</strong></td>
<td>$\frac{2 \cdot h^2}{l^3}$ for $l &gt; h$ (h: thickness)</td>
</tr>
</tbody>
</table>

Table 1. Summary of chord length distributions for various shapes.
The chord length distributions under $\mu$-randomness of some simple shapes are listed in table 1 and shown in fig. 1. These functions are statistically described by the mean chord length $\bar{l}$ and the variance $\sigma^2$. The former can be calculated via the integration technique or with the Cauchy’s equation [43], that is:

$$\bar{l} = \int_0^\infty l \cdot c(l) \cdot dl = \frac{4 \cdot V}{A}$$  \hspace{1cm} (1)

where $V$ is the volume and $A$ is the surface area of the body.

In order to characterize the statistical dispersion of $c(l)$, Kellerer [44] and Rossi [18] introduced the concept of relative variance, defined as the variance normalized to the mean chord length:

$$V_1 = \frac{\sigma^2}{\bar{l}^2}$$  \hspace{1cm} (2)
This quantity permits to quantify the relative contribution of the chord length distribution to the total statistical uncertainty associated to the measured microdosimetric spectra. The values of the mean chord length and of the relative variance are summarized in table 2 for various geometrical shapes. As it can be observed, the sphere has the lowest relative variance (0.125). Thus, it can be considered the optimum shape for microdosimetric measurements, since its track length dispersion contributes slightly to the total uncertainty of the microdosimetric spectra.

<table>
<thead>
<tr>
<th>Shape</th>
<th>Mean chord length</th>
<th>Relative variance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sphere</td>
<td>( \frac{2 \cdot d}{3} ) (d: diameter)</td>
<td>0.125</td>
</tr>
<tr>
<td>Cylinder</td>
<td>( \frac{2 \cdot d \cdot h}{d + 2 \cdot h} ) (d: diameter, h: height)</td>
<td>0.256 for right cylinder (d = h)</td>
</tr>
<tr>
<td>Cube</td>
<td>( \frac{2 \cdot c}{3} ) (c: side length)</td>
<td>0.266</td>
</tr>
<tr>
<td>Infinite Slab</td>
<td>2 \cdot h (h: thickness)</td>
<td>Does not exist</td>
</tr>
</tbody>
</table>

Table 2 Summary of mean chord lengths and relative variances of various shapes.

The hypothesis of infinite straight lines is non realistic, since the interacting particles are always characterized by a finite range. Therefore, a more accurate description of the particle track length distributions should also take into account the particle range distribution. Nevertheless, while the chord length distributions depend on geometrical features only, the knowledge of the particle energy distribution implies a loss of generality, since the spectra of each component of the radiation field should be assessed a-priori.

It should be underlined that this study is limited to microdosimetry of low energy neutron fields. As reported by ICRU report 36 [17], “in the case of neutrons, one deals with the spectra of the initial energy of H ions, He ions and heavier recoils”. By considering the elemental composition of ICRU tissue [45], listed in table 3, the contribution of hydrogen ions to the microdosimetric spectra should be the most important. Moreover, the main interaction of neutrons with hydrogen at low energies is the elastic scattering. Thus, in this preliminary study only recoil-protons were taken into account, for simplicity (see chapter VII for details).
### IV.2. Segment length distributions

Chord length distributions (CLDs) originate when a convex body is exposed to a random distribution of rays with infinite range. If a finite range is considered, the intercept distributions generated are conventionally called “segment length distributions” (SLDs). The analysis of this realistic situation was exhaustively faced by Kellerer [41], who provided an extremely useful procedure for calculating the SLDs given the CLDs. In the following, the formalism developed by Kellerer is described and will be adopted for the evaluation of the SLDs of various shapes.

The procedure makes use of the concept of sum distribution $G(x)$, which is related to the probability density function $g(x)$ through the following equation:

$$G(x) = \int_s^x g(x) \cdot dx$$ \hspace{1cm} (3)

By assuming a sum distribution $C(l)$ of the chord length $l$ (under $\mu$-randomness) and an isotropic field of straight random tracks with a sum distribution $R(u)$ of range $u$, the resulting sum distribution $P(s)$ of the segment length $s$ is given by [ref]:

$$P(s) = \frac{1}{l + u} \cdot \left( C(s) \cdot \int_s^{\infty} R(x) \cdot dx + R(s) \cdot \int_s^{\infty} C(x) \cdot dx \right)$$ \hspace{1cm} (4)
where $\bar{l}$ is the mean chord length given by eq. (1) and $\bar{u}$ is the mean range of the particles generating the tracks. The corresponding SLD can be obtained by differentiating eq. (4), that is:

$$
P(s) = \frac{1}{\bar{l} + \bar{u}} \left( e(s) \cdot \int_{s}^{\infty} R(x) \cdot dx + r(s) \cdot \int_{s}^{\infty} C(x) \cdot dx + 2 \cdot C(s) \cdot R(s) \right)
$$

The mean segment length $\bar{s}$ is given by Kellerer [41] as $(\bar{l}^{-1} + \bar{u}^{-1})^{-1}$. Equation (5) is defined in an uniform and isotropic field of straight tracks and thus, considering the actual experimental situations in microdosimetry, it is theoretically limited to a few experimental cases. Moreover, the secondary charged particles (protons, alphas, etc.) generated in the detector wall by neutron interactions have an anisotropic angular distribution. The symmetry of a spherical detector permits to neglect these problems. For a cylindrical microdosimeter, the minimization of the anisotropy is typically obtained by approximating its sensitive volume to a right cylinder [40].

Therefore, by assuming a detector with an isotropic response, the realistic track length distributions, or SLDs, can be calculated through eq. (5), where the particle range distribution must be accounted for. As mentioned above, the procedure will be limited to the calculation of track distributions of recoil-protons generated via neutron elastic scattering on hydrogen (atomic abundance 63.31% in the ICRU-tissue). Thus, according to the theory of elastic scattering on hydrogen, the energy distribution of recoil-protons produced by neutrons with energy $E_n$ is:

$$
p(E_p) = \frac{1}{E_n} \quad \text{for} \quad 0 \leq E_p \leq E_n
$$

where $E_p$ is the initial energy of recoil-protons (see chapter 3 for the hypothesis). The relation with the probability density function of energy $E_p$ is given by:

$$
p(E_p) \cdot dE_p = r(u) \cdot du
$$

with $r(u)$ the probability density function of the range $u$. Taking into account the expression which relates the stopping power $S_{\text{Tissue}}$ and the range $u$ in tissue:
\[
\frac{dE_p}{du} = S_{\text{Tissue}}^T(E_p)
\]
and by substituting to \(E_p\) the corresponding proton range in tissue, eq. (7) gives:

\[
r(u) = \frac{1}{E_n} \cdot S_{R_{\text{Tissue}}}^T(u)
\]

where \(S_{R_{\text{Tissue}}}^T(u)\) is the stopping power function of range \(u\) in tissue. Fig. 2 shows the probability density function of \(r(u)\) and the sum probability \(R(u)\) at 1 MeV neutron energy.

Through numerical methods, eq. (5) gives the SLDs for a spherical or a cylindrical tissue sensitive volume at the same neutron energy. The results obtained with a unit diameter sphere and a right cylinder of unit dimensions are shown in figs. 3a and 3b, respectively. It should be underlined that these two shapes are characterized by equal values of the mean chord length (table 3) and the mean segment length.
In order to investigate the dependence of SLDs on neutron energy, the mean values \( \bar{s} \) of these distributions were calculated at different neutron energies. As it can be observed in fig. 4, the mean segment length asymptotically tends to the respective CLDs mean chord length (equal to 2/3) for both shapes.

**Fig. 3** Segment length distribution for unit diameter sphere (a) and for a unit dimension right cylinder (b). The range distribution is calculated by eq. (9) for 1 MeV neutron energy

**Fig. 4** Mean segment length for a unit diameter sphere and a right cylinder of unit dimensions at different neutron energies (red line). The mean chord length, equal to 2/3 for both shapes, is also plotted (black line)
IV.3. $\Delta E$ stage track length distributions

A realistic description of the track length distributions of recoil-protons in the $\Delta E$ stage cannot neglect the interaction geometry of neutrons in the converter. A simple analytical model based on a random distribution of the direction of the incoming protons would overestimate the mean track length and the relative variance. In fact, the interaction geometry (i.e. the angular distribution) of the elastic scattering limits the angles of incidence on the $\Delta E$ stage and determines the energy spectrum of the interacting particles. It should be underlined that the following procedure is derived by the analytical model described in chapter III.

For simplicity, stoppers will be distinguished from crossers. At the energy $E_{d,s}^{\Delta E}$ (i.e. the energy deposited by stoppers in the $\Delta E$ stage) stoppers are characterized by track lengths equal to their range in silicon, since they stop inside the $\Delta E$ stage. By contrast, crossers always cross the stage and their track lengths depend only on the emission angle $\theta$.

**Stoppers**

The distributions of the stopper track length can be easily derived by assuming that:

$$s = R^{Si}(E_{d,s}^{\Delta E})$$  \hspace{1cm} (10)

where $s$ is the length of the considered track. At a definite angle $\theta$, the distribution of track lengths $s$ is related to that of the deposited energy $E_{d,s}^{\Delta E}$ in the following way:

$$p_s(s|\cos \theta) \cdot d(s|\cos \theta) = p(E_{d,s}^{\Delta E} | \cos \theta) \cdot d(E_{d,s}^{\Delta E} | \cos \theta)$$  \hspace{1cm} (11)

By considering that, according to eq. 34 chapter III:

$$p(E_{d,s}^{\Delta E} | \cos \theta) = \frac{1}{R^{poly}(E_n)} \cdot \frac{S^{Si}(E^{Si} \left( R^{Si}(E_{d,s}^{\Delta E}) + \frac{a}{\cos \theta} \right))}{S^{poly}(E^{Si} \left( R^{Si}(R^{Si}(E_{d,s}^{\Delta E})) + \frac{a}{\cos \theta} \right))} \cdot \frac{1}{S^{Si}(E_{d,s}^{\Delta E})} \cdot \cos \theta$$  \hspace{1cm} (12)

and, given that
\[
\frac{d(E_{d,s} \Delta E \mid \cos \theta)}{d(s \mid \cos \theta)} = \left( \frac{d(s \mid \cos \theta)}{d(E_{d,s} \Delta E \mid \cos \theta)} \right)^{-1} = \left( \frac{d(R_{s}^{Si}(E_{d,s} \Delta E) \mid \cos \theta)}{d(E_{d,s} \Delta E \mid \cos \theta)} \right)^{-1} = S^{Si}(E_{d,s}^{\Delta E}) \tag{13}
\]

from eqs. (10), (11) and (13):

\[
p_{s}(s \mid \cos \theta) = \frac{1}{R^{poly}(E_{n})} \cdot \frac{S^{Si}\left(E^{Si}\left(s + \frac{a}{\cos \theta}\right)\right)}{S^{poly}\left(E^{Si}\left(s + \frac{a}{\cos \theta}\right)\right)} \cdot \cos \theta \tag{14}
\]

with \(0 \leq s \leq \frac{h}{\cos \theta}\) and \(s \leq R^{Si}(E_{n} \cdot \cos^{2} \theta) - \frac{a}{\cos \theta}\).

Since the elastic scattering gives rise to a probability distribution of the emission angle \(\theta\) which is uniform with respect to \(\cos^{2} \theta\), the probability density \(p(s)\) can be calculated by integrating numerically eq. (14) in the following way:

\[
p_{s}(s) = \int_{\cos(\theta_{\text{max}} \mid s)}^{1} p_{s}(s \mid \cos \theta) \cdot d \cos^{2} \theta \tag{15}
\]

where \(\cos(\theta_{\text{max}} \mid s)\) is given by:

\[
R^{Si}(E_{n} \cdot \cos^{2}(\theta_{\text{max}} \mid s)) = s + \frac{a + h}{\cos(\theta_{\text{max}} \mid s)} \tag{16}
\]

As already mentioned in section III.5, the numerical integration of eq. (15) gives about the same results when the second factor in eq. (14) is evaluated at \(\cos \theta = 1\), that is:

\[
\int_{\cos(\theta_{\text{max}} \mid s)}^{1} \frac{S^{Si}\left(E^{Si}\left(s + \frac{a}{\cos \theta}\right)\right)}{S^{poly}\left(E^{Si}\left(s + \frac{a}{\cos \theta}\right)\right)} \cdot \cos \theta \cdot d \cos^{2} \theta \cong \int_{\cos(\theta_{\text{max}} \mid s)}^{1} \frac{S^{Si}\left(E^{Si}(s + a)\right)}{S^{poly}(E^{Si}(s + a))} \cdot \cos \theta \cdot d \cos^{2} \theta
\]
With the above approximation and by integrating eq. (15), \( p_s(s) \) results:

\[
 p_s(s) = \frac{2}{3} \left[ 1 - \cos(\theta_{\text{max}} | s) \right]^3 \cdot \frac{1}{R_{\text{poly}}(E_n)} \cdot \frac{S^S_i(E^S_i(s + a))}{S^S_{\text{poly}}(E^S_i(s + a))}
\]

for \( 0 \leq s \leq h \), and

\[
 p_s(s) = \frac{2}{3} \left[ \left( \frac{h}{s} \right)^3 - \cos(\theta_{\text{max}} | s) \right]^3 \cdot \frac{1}{R_{\text{poly}}(E_n)} \cdot \frac{S^S_i(E^S_i(s + a))}{S^S_{\text{poly}}(E^S_i(s + a))}
\]

for \( h < s \leq \left( \frac{h}{\cos \Theta_{\text{lim}}} \right) \) \( \ldots (18) \)

Eq. (18) describes the track lengths distribution of stoppers of a single neutron interaction and it represents a component of the complete track length distribution. The behavior of this distribution for different neutron energies is shown in fig. 5.

\[ \text{Fig. 5 Track length distributions of stoppers in the } \Delta E \text{ stage } p_s(s) \text{ at different neutron energies } E_n. \]
Crossers

By considering that crossers are defined as recoil-protons which cross the $\Delta E$ stage, their track length $s$ is simply given by:

$$ s = \frac{h}{\cos \theta} \quad (19) $$

being $\theta$ the emission angle of the crosser. From this expression linking $s$ and $\theta$, it is easy to relate their probability distribution:

$$ p(E_d^E, \cos \theta) \cdot dE_d^E \cdot d \cos^2 \theta = -p_c(E_d^E, s) \cdot dE_d^E \cdot ds \quad (20) $$

where $E_d^E$ is the energy deposited in the E stage by the crosser itself. In fact, at a defined angle $\theta$, the probability to have a crosser track of length $s$ is equal to the probability of reaching the E stage and depositing an energy $E_d^E$. Now, being:

$$ p(E_d^E, \cos \theta) \cdot dE_d^E \cdot d \cos^2 \theta = p(E_p^{\text{int}}, \cos \theta) \cdot dE_p^{\text{int}} \cdot d \cos^2 \theta \quad (21) $$

the probability density of the crosser track length $p_c(s)$ results:

$$ p_c(s) \cdot ds = \left( \int_0^{E_d^E \text{max}} p(E_d^E, s) \cdot dE_d^E \right) \cdot ds = -\left( \int_0^{E_d^E \text{max}} p(E_d^E, \cos \theta) \cdot dE_d^E \right) \cdot d \cos^2 \theta \quad (22) $$

but, from eq. (21) and

$$ \int_0^{E_d^E \text{max}} p(E_d^E, \cos \theta) \cdot dE_d^E = \int_0^{E_d^E \text{max}} p(E_p^{\text{int}}, \cos \theta) \cdot \frac{dE_p^{\text{int}}}{dE_d^E} \cdot dE_p^{\text{int}} = \int_0^{E_d^E \text{max}} \cos \theta \cdot \frac{dR^{\text{poly}} (E_p^{\text{int}})}{d(E_d^E)} \cdot dE_p^{\text{int}} \quad (23) $$

Knowing that:

$$ E_p^{\text{int}} = E^S \left[ R^S (E_d^E) + \frac{a + h}{\cos \theta} \right] \quad (24), $$
therefore:

\[
\int_0^{E_d^{E_{\text{max}}}} p(E_d, \cos \theta) \cdot dE_d = \frac{R_{\text{poly}} \left( E_{\text{Si}}(R_{\text{Si}}(E_d^{E_{\text{max}}}) + \frac{a + h}{\cos \theta}) \right)}{R_{\text{poly}}(E_n)} \cdot \cos \theta
\]

(25)

Given that (see eq. (19)):

\[
\frac{d \cos^2 \theta}{ds} = \frac{d \left( \frac{h^2}{s^2} \right)}{ds} = -\frac{2 \cdot h^2}{s^3}
\]

(26)

and

\[
R_{\text{poly}} \left( E_{\text{Si}} \left( R_{\text{Si}} \left( E_d^{E_{\text{max}}} \right) + \frac{a + h}{\cos \theta} \right) \right) = R_{\text{poly}} \left( E_n \cdot \cos^2 \theta \right)
\]

(27)

the probability density function of crosser track length \( s \) normalized to unit neutron interaction is:

\[
p_c(s) = \frac{R_{\text{poly}} \left( E_n \cdot \frac{h^2}{s^2} \right) - R_{\text{poly}} \left( E_{\text{Si}} \left( \frac{a + h}{h} \cdot s \right) \right)}{R_{\text{poly}}(E_n)} \cdot 2 \cdot \frac{h^3}{s^4}
\]

(28)

with \( h < s \leq \left( \frac{h}{\cos \Theta_{\text{lim}}} \right) \).

Alternatively, this probability can be calculated by considering the polyethylene-equivalent layer of interest at the emission angle \( \theta \) normalized to the maximum layer of interest at \( E_n \) (see chapter III).

Therefore, the total track length distribution can be derived by summing, in the proper interval, eq. (18) and eq. (28) for each \( s \) value. The results of this analytical procedure are shown in fig 6 for different neutron energies. The maximum track length \( s_{\text{max}} \) at each neutron energy is indicated in the same figure.

Fig. 7a shows the dependence of the mean track length on different neutron energies. It should be noted that the mean track length depends slightly on the neutron energy above 1 MeV. In the same figure, the maximum track length \( s_{\text{max}} \) is plotted at different neutron energies. As can be observed, \( s_{\text{max}} \) is lower than 10 \( \mu \text{m} \) even for 10 MeV neutrons. This is due to the interaction geometry of the elastic scattering that limits the angles of incidence on the \( \Delta E \) stage. As a confirmation, the mean segment length of a slab under \( \mu \)-randomness was
calculated by following Kellerer’s procedure (see eq. (5)). The results are plotted in fig. 7b for different neutron energies. The mean value calculated with Kellerer’s model is always higher than that evaluated by considering the angular distribution of the elastic scattering, according to the remarks stated above.

Fig. 6 Total track length distributions in ΔE stage for different neutron energies. The maximum track length is also calculated for each energy.

Fig. 7 (a) Mean and maximum track lengths in ΔE stage for different neutron energies $E_n$. (b) Comparison between the mean track length in ΔE stage calculated by considering the actual interaction geometry and mean track length obtained from Kellerer’s procedure (eq. 5) for an infinite slab of thickness equal to that of ΔE stage.
IV.4. Microdosimetric shape equivalence criteria

Kellerer [40] proposed the dose-mean energy imparted per event \( \overline{\varepsilon}_D \) for comparing the performance of microdosimetric shapes. This quantity is closely related to the dose-mean lineal energy by the following expression:

\[
\overline{\varepsilon}_D = \frac{\overline{\varepsilon}_D}{1}
\]  

(29)

In general \( \overline{\varepsilon}_D \) is calculated from the dose-distribution of the energy imparted per event in the following way:

\[
\overline{\varepsilon}_D = \frac{\int_0^\infty \varepsilon^2 \cdot p(\varepsilon) d\varepsilon}{\int_0^\infty \varepsilon \cdot p(\varepsilon) d\varepsilon}
\]

(30)

By assuming a constant linear energy transfer \( L \), the energy imparted \( \varepsilon \) is simply given the product of \( L \), the energy deposited per unit path length, and \( s \), the track length, thus:

\[
\varepsilon = L \cdot s
\]

(31)

From eqs. (30) and (31), the dose-mean energy imparted per event can be calculated with the following equation:

\[
\overline{\varepsilon}_D = L \cdot \frac{\int_0^\infty s^2 \cdot p(s) ds}{\int_0^\infty s \cdot p(s) ds}
\]

(32)

where \( p(s) \) is the probability density distribution of a track of length \( s \) and \( \overline{s} \) is the mean value of \( p(s) \). To equate \( \overline{\varepsilon}_D \) for the two different shapes corresponds to equate the second member of eq. (32). Thus, the following parameter can be defined:

\[
\overline{s} = \frac{\int_0^\infty s^2 \cdot p(s) ds}{\int_0^\infty s \cdot p(s) ds} = \frac{\overline{s}^2}{\overline{s}}
\]

(33)
where $\bar{S}$ is the mean track length and $\bar{S}^2$ is its second moment. Fig. 8 shows the values assumed by $\bar{S}$ at different neutron energies and for different volume shapes. Therefore, according to Kellerer’s suggestion [40], two different shapes are considered equivalent from a microdosimetric point of view when the parameters $\bar{S}$ of their track length distributions are equal.

Fig. 8 Parameter $\bar{S}$ versus neutron energy for (a) a unit diameter sphere, (b) a right cylinder of unit dimensions and (c) the $\Delta E$ stage.
Chapter V

CHARACTERIZATION OF THE MONOLITHIC SILICON TELESCOPE

The feasibility studies of a microdosimeter and a neutron spectrometer based on silicon devices (see chapters I and II) allowed to individuate the problems and the limitations for the utilization of simple photodiodes in these applications.

The preliminary investigation on a silicon microdosimeter realized with a pin diode highlighted the need of minimizing the so-called “field-funneling effect” (FFE) [27]. As already mentioned, this effect is due to a local distortion of the electric field at the edge of the depletion region, leading to the collection of charge from the non-depleted zone. Therefore, the sensitive thickness of the diode depends on the interacting particle LET. Since microdosimetry requires a well-defined sensitive volume, such a detector cannot be used for this application. This conclusion was in agreement with other works, where similar devices were studied and tested [22-25]. Rosenfeld and collaborators proposed a new device realized with the Silicon-On-Insulator (SOI) technology (see chapter II) in order to confine the charge collection process by exploiting the presence of an insulator (SiO$_2$) at the edge of the sensitive layer [21,29]. In the framework of the collaboration of the Nuclear Engineering Department with the ST-microelectronics, a different detector was investigated: the monolithic silicon telescope [47,48]. This choice was strictly related to the peculiar structure of the silicon device, which allows not only to minimize the field funneling effect, but also to optimize the microdosimetric measurements by providing information about the energy and type of the interacting particles.

As already mentioned, the telescope can be applied both in silicon microdosimetry and in neutron spectrometry. For the latter application, a feasibility study of a recoil-proton spectrometer based on a silicon diode [10] recommended a further reduction of the minimum detectable energy. This minimum detectable energy is imposed by a field of secondary electrons which overwhelms the recoil-proton spectrum below a few MeV (1.7 MeV for the diode accounted for in ref. [10]). Secondary electrons are generated by the interactions of the photons associated to the neutron field with the structural materials of the detector assembly.
Pulse shape discrimination was applied to the electronic signals generated by a PIN diode for eliminating the secondary electron contribution [11]. In this way that detection system was able to measure neutron spectra above 1.2 MeV.

The monolithic silicon telescope was designed for measuring the energy and the type of the interacting particles. This feature suggested to investigate its performances for discriminating electrons from recoil-protons and therefore to further reduce the minimum detectable energy when applied as a neutron spectrometer. The energy deposited in the \( \Delta E \) stage of the silicon telescope is a LET-related quantity. Thus, the capability of measuring this quantity event-by-event was exploited for realizing a particle discrimination which is effective down to about 100 keV.

### V.1 The monolithic silicon telescope: C-V measurements

The monolithic silicon telescope (sketched in fig. 1) is characterized by two different stages: a thin \( \Delta E \) stage and a thick \( E \) stage. The former is about 2 \( \mu \)m in thickness, while the latter is about 500 \( \mu \)m thick. The two stages are realized on a single silicon wafer and share a \( p^+ \) well which is realized through a deep ion implantation. The charges generated in silicon by the incident radiation are collected separately by the two stages and the highly-doped \( p^+ \) cathode acts as a “watershed” for this separation. As shown in fig. 2, the steep doping profile, realized through implantation of 1.25 MeV boron ions, should guarantee the minimization of the “field funneling effect”, by confining the \( \Delta E \) stage and limiting the edge fluctuations.
An accurate characterization of the electrical properties of the two stages of the telescope was performed in order to investigate their geometrical structure in static conditions.

Capacitance measurements of the $\Delta E$ and E stages were performed for devices with different sensitive areas, namely 1, 10 and 25 mm$^2$. The E stage is uniformly doped and the dependence of its capacitance on reverse bias voltage is described by the simple step-junction model. The depletion voltage of this stage resulted to be about 140 V. For applied bias voltage above this value the entire thickness, 500 $\mu$m, participates to the charge collection. The doping profile across the $\Delta E$ stage, on the contrary, is non-uniform and the features of this stage were investigated by an analytical procedure. The result of the C-V measurement is shown in fig. 3a, where the specific capacitance of the $\Delta E$ stage is plotted versus the reverse bias voltage. The depletion layer thickness can be derived by the relation between the specific junction capacitance $C_j$ and the depletion layer thickness $w_{dl}$.

\[ C_j = \frac{\varepsilon_{Si}}{w_{dl}} \]  

(1)
where $\varepsilon_{Si}$ is the silicon dielectric constant. As it can be observed in Fig. 3b, the depletion region width of the $\Delta E$ stage is always below 1 $\mu$m and it slightly depends on the bias voltage.

![Fig. 3](attachment:image.png)

**Fig. 3** (a) Specific capacitance of the $\Delta E$ stage of the monolithic silicon telescope at different reverse bias voltages. (b) Depletion layer width of the $\Delta E$ stage versus reverse bias voltage.

In order to verify the characteristics of this stage, a general model based on Poisson’s equation in one dimension was developed:

$$\frac{d^2 \phi(x)}{dx^2} = -\frac{\rho(x)}{\varepsilon_{Si}}$$

(2)

where $\phi(x)$ is the electric potential and $\rho(x)$ is the net charge density distribution in $\Delta E$ stage. This distribution can be calculated from the doping profile shown in fig. 2, fixing at $x = 0$ the metallurgical junction where $\rho(x)$ is zero. It should be underlined that the dopant distribution reported in fig. 2 was provided by ST-Microelectronics and it is the result of process simulations. Now, within the interval (-a –b), $\rho(x)$ is expressed by:

$$\rho(x) = \begin{cases} 
q \cdot N_D(x) & \text{if} \ (-a \leq x \leq 0) \\
-q \cdot N_A(x) & \text{if} \ (0 < x \leq b)
\end{cases}$$

(3)

where $N_D(x)$ and $N_A(x)$ are the arsenic and boron concentration in the $\Delta E$ stage, respectively.
The trend of $\rho(x)$ is shown in fig. 4a. By integrating eq. (3) and by applying the boundary condition that the electric field $E(x)$ must vanish at both edges $X_n$ and $X_p$ of the depleted region:

$$E(x) = \begin{cases} \frac{x \cdot q \cdot N_D(x)}{\varepsilon_{Si}} \cdot \frac{dx}{x} & \text{if} \quad (X_n \leq x \leq 0) \\ \frac{x \cdot q \cdot N_A(x)}{\varepsilon_{Si}} \cdot \frac{dx}{x} & \text{if} \quad (0 \leq x \leq X_p) \end{cases}$$ (4)

The corresponding shape of the electric field is shown in fig. 4b. Another integration will now yield the electric potential $\phi(x)$. Even in the absence of any external voltage, there is a potential difference between the edges of the depleted region which is equal to the built-in voltage $\phi_{bi}$. Thus, by applying an external bias voltage $V$, the boundary conditions give:

$$\phi(X_n) = \phi_{bi} + V \quad \text{and} \quad \phi(X_p) = 0$$ (5)

Therefore:

$$\phi(x) = \begin{cases} \phi_{bi} + V + \int_{x}^{x_n} E(x) \cdot dx & \text{if} \quad (X_n \leq x \leq 0) \\ \int_{x}^{x_p} E(x) \cdot dx & \text{if} \quad (0 \leq x \leq X_p) \end{cases}$$ (6)

The corresponding shape of the electric potential is shown in fig. 4c. From the general model of the p-n junction with no externally applied voltage:

$$\phi_{bi} = \frac{k_B \cdot T}{q} \cdot \ln \left( \frac{N_D(X_n) \cdot N_A(X_p)}{n_i^2} \right)$$ (7)

where $k_B$ is the Boltzmann constant, $T$ is the absolute temperature and $n_i$ represents the intrinsic carrier density. The continuity conditions at $x = 0$ of the solutions given by eqs. (4) and (6) and from eq. (7) give the following system of two equations:
with $\phi_{bi}$ evaluated for $V = 0$ only. The values of $X_n$ and $X_p$ at any bias voltage $V$ can be obtained numerically from eq. (8).

This numerical model permits to evaluate the transition region extension on both sides of the junction and thus to estimate the width and the localization of the depletion layer. The solutions at $V = 5$ V gives a depletion layer width equal to about 0.9 $\mu$m, in agreement with the measured value (fig. 3b).
The calculation of the depletion layer thickness at different bias voltages allows to deduce the theoretical C-V curve (by exploiting eq.(1)). The results of this numerical procedure is compared with the C-V measurement of ΔE stage with different sensitive areas in fig. 5. The discrepancies (of the order of a few percent at the lower bias voltages) may be due to inaccuracies in the simulated doping profile provided by ST-Microelectronics. These discrepancies result to be more significant at the equilibrium condition (i.e. at the built-in). However, at operating bias voltages above 2 V, the agreement is fairly good, confirming the validity of the dopant distribution in proximity of the boron peak.

![Graph showing C-V characteristics](image)

**Fig. 5** Comparison between results of capacitance measurements and analytical model based on simulated doping profile.

### V.2 Noise characterization and remarks

The specific capacitance of the ΔE stage is of the order of 10 nF cm\(^{-2}\) (fig. 3a). According to the capacitance-thickness relation given by expression (1), this high value is mainly due to the micrometric thickness of the charge space region of this stage. Since high capacitance values correspond to poor energy resolutions, some criteria must be taken into account in order to optimize the performance of the detection system.
In a spectroscopic system, the noise is usually expressed in terms of Equivalent Noise Charge (ENC), whose dependence on the detector specific capacitance $C_{det}^0$ and on the leakage current density $J_{leak}^0$, by neglecting the 1/f noise, is given by the following formula:

$$ENC^2 = \left( \frac{k_B \cdot T \cdot m}{\omega_T} \right) \cdot C_{det}^0 \cdot A \cdot \frac{1}{\tau_{sh}} + q \cdot J_{leak}^0 \cdot A \cdot \tau_{sh}$$

where $m$ is a matching factor between the detector and the input transistor capacitance $[49]$, $\omega_T$ is the JFET unity gain angular frequency, $\tau_{sh}$ is the shaping time of the spectroscopy amplifier filter and $A$ is the detector area. For the case of a point source located at a distance $d$ such that $d^2 >> A$, the absolute efficiency is proportional to detector area. Therefore, an appropriate choice of the detector area must trade off the specifications about the efficiency and the energy resolution (proportional to $ENC^2$).

![Figure 6](image.jpg)

**Fig. 6** Noise characteristic of the $\Delta E$ stage (sensitive area $10 \text{ mm}^2$). The optimum shaping time is about 20 $\mu$s and the corresponding resolution is about 10 keV FWHM-Si
To face this problem, a custom low-noise front-end electronics was developed. Discrete JFET were characterized and tested in order to individuate the optimum transistor for the considered application. Accurate noise measurements were carried out for a device characterized by a sensitive area of 10.35 mm$^2$. The $\Delta E$ stage of this monolithic silicon telescope, marked by a junction capacitance of about 1.2 nF at 5 V, was coupled to a high capacitance JFET (INTERFET NJ3600) with an input capacitance of about 550 pF. The results are shown in fig.6. The optimum shaping time is 20 µs for which the best resolution of 10 keV FWHM-Si is obtained. The measured noise resulted to be much higher than the computed one, plotted with the continuous line in the same figure. To investigate this behavior, the detector was replaced by a capacitor of equal capacitance (1.2 nF) and the noise was measured again. In this configuration the agreement with the theory is good and this indicates that the additional noise source should be ascribed to the detector, in particular to the presence of a parasitic series resistance. In fact, by observing the forward I-V characteristic of the biased detector, shown in fig. 7, a deviation from the theoretical exponential growth, due to a series resistance of about 7.4 $\Omega$, can be recognized.

![I-V characteristic of the biased $\Delta E$ stage. The deviation from an exponential growth is given by a series resistance $Rs$.](image)
A detailed description of the noise characterization would require a long analysis and it will not be presented here. It should be concluded that the best compromise between energy resolution and efficiency can be obtained by using the telescope with a sensitive area of 1 mm$^2$. For such a detector the capacitance is about 160 pF and the electronic noise is about 7 keV FWHM-Si at 10 µs shaping time. While the noise of the ΔE stage is a serious concern due to the micrometric thickness of the depletion layer, the specific capacitance of the fully depleted E stage is small (of the order pF cm$^{-2}$) and the energy resolution is quite good.

V.3 Characterization of the sensitive thickness

The depleted layer of the ΔE stage is about 0.9 µm thick at 5 V. This evaluation was performed through CV measurements, i.e. by a small dynamic perturbation of a static biasing. As already mentioned, when a charged particle interacts with the detector, the field funneling effect could alter the boundaries of the depleted region, determining a corresponding sensitive thickness higher than the depleted one. Thus, the accurate estimation of the ΔE stage effective thickness requires the irradiation of the silicon telescope with charged particles.

![Scatter plot acquired by irradiating the monolithic silicon telescope with a polychromatic alpha source. $E_d^{\Delta E}$ and $E_d^{\text{tot}}$ are the energy deposited in the ΔE stage and in ΔE plus E stage.](image)
A dedicated setup was realized in which a polychromatic $\alpha$ source of $^{241}$Am illuminates the detector through a collimator that selects the particles normally impinging on the central part of the diode. Signals from both the $\Delta E$ and $E$ stages were collected in the coincidence mode and analyzed with a multi-parametric multi-channel analyzer. The scatter plot of the energy deposited in the $\Delta E$ stage, $E_d^{\Delta E}$, versus the total energy deposited in the two stages, $E_d^{\text{tot}}$ (sum of $\Delta E$ and $E$ signals) is shown in fig.8. The maximum energy of the alpha particles is about 3.5 MeV due to their slowing down in air (the irradiation distance was about 1 cm) and in the dead layer of the silicon telescope. However, the energy $E_d^{\Delta E}$ shows a typical dependence on the inverse of $E_d^{\text{tot}}$ above about 550 keV. Below this energy, the particles stop in the $\Delta E$ stage and $E_d^{\Delta E}$ is equal to $E_d^{\text{tot}}$.

The quantitative investigation of the experimental results was performed by exploiting an analytical procedure based on the model described in chapter III. The aim was to calculate the theoretical energy deposited in a $\Delta E$ stage of arbitrary thickness, at different energies of a given particle type (protons, alphas, etc.).

By assuming a particle direction perpendicular to the detector surface, the range in silicon at the energy $E_i$ is equal to the sum of the $\Delta E$ stage thickness $h$ and the range in the $E$ stage $R_{Si}(E_d^E)$, that is:

$$R_{Si}(E_i) = h + R_{Si}(E_d^E) \quad (10)$$

where $E_d^E$ is the energy deposited in the $E$ stage. By inverting this equation, the dependence of the energy $E_d^E$ on the particle energy $E_i$ results to be expressed by the following equation:

$$E_d^E = E^{Si}(R^{Si}(E_i) - h) \quad (11)$$

where the function $E^{Si}$ is the inverse function of $R^{Si}$. The energy deposited in the $\Delta E$ stage $E_d^{\Delta E}$ is given by the difference between the particle energy $E_i$ and the energy deposited in the $E$ stage (eq. (11)), thus:

$$E_d^{\Delta E} = E_i - E^{Si}(R^{Si}(E_i) - h) \quad (12)$$
for $R^\text{Si}(E_i) \geq h$, i.e. the particle is a crosser. If the particle stops inside the $\Delta E$ stage, the energy deposited is exactly equal to the energy of the incoming particle at the entrance of the $\Delta E$ stage. It corresponds to the total energy measured within the entire telescope, regardless the presence of a surface dead layer.

In general, the doping profile realized to create two separated stages ($\Delta E$ and $E$) through deep boron implantation (see fig. 2) could give rise to an intermediate layer at the $\Delta E$–$E$ interface where the charge collection process is dominated by diffusion. This can determine the presence of a deep dead layer between the two stages of the telescope.

In this case, the energy $E_d^{\text{tot}}$ results to be lower than the particle energy $E_i$, since a fraction of $E_i$ is lost in this deep dead layer. By exploiting the same analytical procedure described above, the total energy deposited in the telescope depends on $E_i$ in the following way:

$$E_d^{\text{tot}} = E_d^{\Delta E} + E_d^E = E_i - E^\text{Si}(R^\text{Si}(E_i) - h) + E^\text{Si}(R^\text{Si}(E_i) - h - a_{\text{dd}})$$  \hspace{1cm} (13)

for $R^\text{Si}(E_i) > (h + a_{\text{dd}})$, where $a_{\text{dd}}$ is the deep dead layer thickness. Of course, if $h \leq R^\text{Si}(E_i) \leq (h + a_{\text{dd}})$, $E_d^{\text{tot}} = E_d^{\Delta E}$ and, again, if $R^\text{Si}(E_i) \leq h$, $E_d^{\text{tot}} = E_i$. Thus, using eqs. (12) and (13) as equations of an implicit function, one can evaluate the realistic relation between the energy deposited in the $\Delta E$ stage and the total energy deposited in the telescope stages.

![Diagram](image-url)  

**Fig. 9** Trend of the energy deposited in a $\Delta E$ stage of thickness $h$ by alpha particles of different incident energies $E_i$: results of analytical model (solid lines) and Monte Carlo simulations with the SRIM code (scatters).
Fig. 9 shows the results of the analytical procedure applied to the case of the alpha-particle irradiation, together with those obtained from Monte Carlo simulations with the SRIM code [32,33]. In this example, the energy-range relations are taken from SRIM tables and the presence of a deep dead layer was neglected. The good agreement (discrepancies below 1 %) between the simulation and the analytical procedure can be considered as a validation of the model described above. This simple model was adopted to elaborate the experimental results shown in figure 8. The deep dead layer was assumed to be negligible. The analytical relation between $E_d^{\Delta E}$ and $E_d^{\text{tot}}$ was fitted to the measured scatter plot (by neglecting the uncertainties) by adjusting the $h$ value in eq. (12). The aim was to determine the effective extension of the charge collection region in the $\Delta E$ stage. As shown in fig 10, the effective sensitive thickness $h$ resulted to be included between 1.85 $\mu$m and 1.95 $\mu$m.

![Graph showing experimental scatter plot obtained by alpha particles irradiation and analytical results](image)

**Fig. 10** Experimental scatter plot obtained by alpha particles irradiation and analytical results (eq. 12). The effective sensitive thickness $h$ resulted to be included between 1.85 $\mu$m and 1.95 $\mu$m.

A more accurate procedure was carried out by analyzing directly the experimental scatter plot. From eq. (12), the following relation can be obtained:
\[ h = R^{Si}(E_i) - R^{Si}(E_i - E_{dE}) \] (14)

At a definite energy \( E_i \), this theoretical relation allows to calculate the sensitive thickness \( h \) for a given value of the deposited energy \( E_{dE} \). Actually, as it can be observed in [Fig. 10], each \( E_i \) corresponds to a distribution of \( E_{dE} \) values. This spread is due to the statistical uncertainties related to both the radiation interaction and the detection processes, mainly due to charged particle straggling and electronic noise. As it is well known, the electronic noise can be described by a Gaussian distribution.

Straggling can be described by different distributions, depending on the mean number of ionizations promoted by the interacting charged particle within the detector sensitive layer. The description of ionization fluctuations is characterized by the significance parameter \( \kappa \), defined as the ratio of mean energy loss \( \xi \) to the maximum allowed energy transfer in a single collision with an atomic electron [50-52], i.e.

\[ \kappa = \frac{\xi}{E_{MAX}} \] (15)

where

\[ E_{MAX} = \frac{2 \cdot m_e \cdot \beta^2 \cdot \gamma^2}{1 + 2 \cdot \gamma \cdot \frac{m_e}{m_x} + \left( \frac{m_x}{m_e} \right)^2} \] (16)

with \( \gamma = E / m_x \), \( E \) is energy and \( m_x \) the mass of the incident particle, \( \beta^2 = 1 - 1/\gamma^2 \) and \( m_e \) is the electron mass. From the Rutherford scattering cross section, for an incident particle of charge \( z \) that traverses an absorber element of material with density \( \rho \), atomic number \( Z \) and atomic weight \( A \):

\[ \xi = \frac{2 \pi \cdot z^2 \cdot e^4 \cdot N_{Av} \cdot Z \cdot \rho \cdot \delta x}{m_e \cdot \beta^2 \cdot c^2 \cdot A} \] (17)

where \( N_{Av} \) is Avogadro’s number and \( \delta x \) is the thickness of the material.

Depending on the \( \kappa \) value, the following distributions can be adopted [ref]:
- for \( \kappa < 0.01 \), Landau distribution;
- for \( 0.01 < \kappa < 10 \), Vavilov distribution;
- for \( \kappa > 10 \), Gaussian distribution, according to the central limit theorem.
Without going deeply into this matter, it should be underlined that in the present case \( \kappa \) is always higher than about 37. Therefore the ionization fluctuations are well described by a Gaussian distribution.

In the experimental scatter plot, each \( E_i \) corresponds to a Gaussian distribution of \( E_d^{\Delta E} \) values. Thus, the distribution of the mean \( E_d^{\Delta E} \) values can be obtained at the different alpha energies by fitting this distribution at each \( E_i \) values. The results of this fitting procedure are shown in fig. 11. Now, through eq. (14), it is easy to calculate the value \( h \) corresponding to each \( E_i \) value. As it can be observed in fig. 12a, apart from the statistical uncertainties, the \( h \) values slightly depends on the alpha energy, or on the alpha LET. This is a demonstration that the \( \Delta E \) stage of the monolithic silicon telescope is able to minimize the field funneling effect.

The \( h \) values calculated with eq. (14) can be collected in a Gaussian-like frequency distribution, as it is shown in fig. 12b. The parameters of this last Gaussian function give the best estimation of the actual \( \Delta E \) stage sensitive thickness, which results to be about \( 1.88 \pm 0.01 \) \( \mu \)m. This value, calculated with alpha irradiations, completely differs from the depletion
layer thickness that was estimated from CV measurements. As it can be observed in fig. 2, it corresponds approximately to the peak of the doping profile. It should be noted that the standard deviation of the \( \Delta E \) stage thickness (0.01 \( \mu m \)) was calculated from the gaussian distribution of the average values of \( h \) in turn evaluated from the gaussian distribution of the values corresponding to each \( E_i \) value of the scatter-plot.

![Fig. 12](image)

**Fig. 12** (a) Estimated \( \Delta E \) stage thickness \( h \) at each \( E_{i \text{tot}} \) value. (b) Frequency distribution of the \( h \) values (histogram) and the corresponding Gaussian function fit (black line). The Gaussian distribution is characterized by a mean value of 1.88 and a standard deviation of about 0.01.

The experimental results emphasized that on the one hand the distortion of the depleted region owing to the field funneling effect is present, but on the other hand, that the \( \Delta E - E \) structure is capable of confining the charge collection in the \( \Delta E \) stage at a definite value through the deep p+ cathode. The charges generated above the p+ well participate to the formation of the \( \Delta E \) signal, while those created below it contribute to the generation of the E signal, regardless the interacting particle energy. Moreover, the good agreement between analytical and experimental results (see fig. 10) demonstrates that there is no dead layer between the \( \Delta E \) and the E stage of the telescope, since this deep dead layer was assumed to be negligible in the analytical procedure.
V.4 Ion beam analysis: charge collection and sensitive volume characterization

As already mentioned, one of the most important requirements of a solid state microdosimeter is the uniformity of its sensitive thickness. This implies the minimization of the field funneling effect (see chapter II), thus the capability of keeping the $\Delta E$ stage boundaries at a fixed position with respect to the charge collection process. This requirement also involves the uniformity of the overall geometrical structure and of the fabrication parameters, such as implantation, metallization, dead layers, etc.

An accurate investigation of this characteristic was carried out through ion beam analysis (IBA), in collaboration with prof. Rosenfeld, University of Wollongong, Australia [53, 54]. This technique is based on the irradiation of the detector with a microbeam of monoenergetic ions. The IBA permits to map the charge collection over the entire sensitive area by acquiring the signals generated in the two stages of the silicon telescope for each impinging particle. The IBA was performed at the Australian Nuclear Science and Technology Organization (ANSTO) by using an ion probe of alpha-particles with a selectable micrometer spot size. The acquisition system allowed to acquire and process, after an accurate calibration, the signals generated in the detector stages together with the x-y positions of the impinging particles over different portions of sensitive area. Since the minimum spot size was about 1 $\mu$m, the charge collection process could be analyzed accurately in the border zones, also investigating the functionality of the guard rings for the sensitive volume confinement. It should be underlined that the results of these measurements are still in process and therefore only some preliminary considerations will be presented here.

![Fig. 13 The device and the various scan orientations adopted for IBA with 3 MeV alpha-particles. Only the results of scans A and B are presented.](image)
Fig. 13 shows the device and the various scan orientations adopted for IBA with 3 MeV alpha-particles. In the following, only the results of scans A and B will be presented. Scan incorporates the contacts from the p+ cathode (lower) to the ring (upper) while scan B is focused on the n+ anode. The results of scan A are shown in fig. 14. This figure reports both the x-y mosaic and the deposited energy distribution of the events generated in the ΔE stage. The mean energy deposited in the ΔE sensitive volume $E_{\Delta E}$ (corresponding to the peak energy in fig. 14b by 3 MeV alphas resulted to be about $375 \pm 13$ keV (FWHM = 30 keV).

By considering that the energy lost by the incident 3 MeV alpha particles in the titanium dead layer can be estimated of about 80 keV, the thickness $h$ of the ΔE stage can be estimated with expression (15) for $E_{\Delta E} = 375$ keV (see above) and $E_i = 2.920$ MeV (the energy of the alpha-particles at the entrance of the ΔE stage). The standard deviation $\sigma_h$ of this thickness can be calculated through the uncertainty propagation law:

$$\sigma_h = \left( \frac{\partial h}{\partial E_{\Delta E}} \right)^2 \cdot \sigma_{E_{\Delta E}}^2 = \left( \frac{\partial}{\partial E_{\Delta E}} \left( R^{Si} (E) - R^{Si} (E - E_{\Delta E}) \right) \right)^2 \cdot \sigma_{E_{\Delta E}}^2 = \frac{\sigma_{E_{\Delta E}}}{S_u^{Si} (E - E_{\Delta E})}$$

where $\sigma_{E_{\Delta E}}$ is the standard deviation of $E_{\Delta E}$ and $S_u^{Si}$ is the stopping power in silicon of an alpha particle. The uncertainty associated to the incoming beam energy $E$ was neglected in the expression above. The ΔE thickness results to be about $1.88 \pm 0.06$ μm, according to the value estimated from the scatter plots of an isotopic (and polychromatic) alpha source. Furthermore, the contribution of low energy events (edge at 250 keV fig.14) is probably due to the
channeling of alpha-particles, since, as can be observed in fig. 15, for each event the amount of energy which is missing from the $\Delta E$ stage is deposited in the $E$ stage. This effect is signaled by the distortion at the right hand (higher energies) of the full energy peak of the $E$ stage acquired during scan A (fig. 15).

![Fig. 15 Results of scan A. X-y mosaic (a) and deposited energy distribution (b) of the events generated in the $E$ stage.](image1)

Scan B (fig. 16) was focused on the $n^+$ anode. An additional peak at about $160 \pm 5$ keV can be observed in the metallization region when the x-y mosaic is compared to the distribution of energy deposited in the $\Delta E$ stage. This peak may be due to the lower thickness of the $\Delta E$ stage below the bonding pad. The effective sensitive thickness of this zone was estimated to be $0.83 \pm 0.03 \mu$m. However, it should be underlined that this unwanted effect can be avoided by excluding this zone from the irradiated area.

![Fig. 16 Results of scan B. X-y mosaic (a) and deposited energy distribution (b) of the events generated in the $\Delta E$ stage.](image2)
As already described in chapter I, the feasibility of measuring neutron spectra with silicon devices directly coupled to a plastic converter has been demonstrated in ref. [10]. In that work, a 300 µm thick silicon diode with a sensitive area of about 3 mm² was covered with a polyethylene layer in order to realize a recoil-proton spectrometer. The detection system was characterized with monoenergetic neutron fields. Below about 1.7 MeV, the measured response functions resulted to be completely overwhelmed by the contribution of the photons associated to the neutron field. Therefore, pulse shape discrimination was employed to reduce the minimum detectable energy of the spectrometer [11]. The silicon PIN diode, irradiated in the “reverse-injection” configuration (i.e. by placing the polyethylene converter in contact with the N⁺ layer), was connected to a fast, low-noise electronic chain capable of providing the collection time of the charge generated in the detector by the incident radiation. The aim was to eliminate the contribution of photons to the measured recoil-proton spectra exploiting the difference in the rise-time of signals generated by radiations of different LET. Through this method, it was realized a neutron spectrometer capable of measuring energy spectra from about 1.2 MeV up to 6 MeV. While the higher detection limit is due to the sensitive thickness of the diode, the minimum detectable energy is limited by the adopted pulse shape discrimination technique which is less effective below 1.2 MeV especially in the presence of intense photon fields.

Therefore, the realization of a neutron spectrometer based on a silicon device operating over a wide energy range is unavoidably related to the features of gamma-neutron discrimination at low energies, in particular below 1 MeV.

As already mentioned in chapter IV, the monolithic silicon telescope, manufactured by ST-Microelectronics for measuring charged particles produced by ion-induced nuclear reactions, can provide the simultaneous evaluation of the energy and type of an interacting radiation [47,48]. Therefore, it could represent a valid alternative to pulse shape discrimination. Although it was designed for heavy charged particle identification, its
capability of discriminating light particles, such as protons and electrons, was not investigated before. A set of telescopes mounted on a shell surrounding the target is used for measuring secondary charged particles from ion-induced reactions. In this application, the source (i.e. the target) position is well known and the surface of each telescope detector faces the target. When the telescope is exposed to a mixed neutron-photon field, a more complex situation has to be handled. The distributions of the energy and of the incident direction of secondary electrons is unknown a priori and could imply an energy deposition similar to that generated by recoil-protons. It should also be underlined that the high electronic noise which characterizes the $\Delta E$ stage can reduce the efficiency of the discrimination.

The following sections describe the characterization of a neutron spectrometer based on a monolithic silicon telescope. The experimental data analysis of the irradiations with monoenergetic neutrons was supported with the analytical models described in chapter III. The reconstruction of continuous neutron spectra is based on an iterative unfolding procedure. The unfolded spectra were compared with literature data acquired through time of flight technique.

### VI.1. Experimental set-up

The neutron spectrometer was realized by placing a 1 mm thick polyethylene layer in contact with the monolithic silicon telescope. This detector, characterized by the geometrical structure shown in fig. 1, accomplishes the role of detector of the recoil-protons generated in the converter by neutrons impinging on hydrogen nuclei. Since the presence of recoil-carbon ions can be neglected (due to their mass and atomic number, as explained in chapter III), the spectra acquired by the silicon detector correspond to the distribution of energy deposited within the sensitive volume by recoil-protons only (apart from the secondary electrons mentioned in the previous section).

![Fig. 1 Sketch of the monolithic silicon telescope](image-url)
The sensitive area of the tested detector is about 1 mm$^2$. Both the ΔE and the E stage, (thickness 2 µm and 500 µm, respectively), were biased and two different electronic chains were utilized to amplify and shape the signals induced by the interacting radiation.

The ΔE stage was characterized by a detector capacitance of about 165 pF at the operative bias voltage of 5 V. It was connected to a JFET (INTERFET-J) with an input capacitance of about 3 pF and to a commercial preamplifier (Amptek-A250). The signals generated in the ΔE stage were amplified through a standard module (Ortec 672), with a shaping time of about 3 µs, obtaining an electronic noise of about 7 keV FWHM-Si. The E stage is characterized by a detector capacitance of about 6 pF at the operative bias voltage of 120 V. It was connected to a JFET (IF-140) with an input capacitance of about 3 pF and to a commercial preamplifier (Amptek-A250). The amplifier (Ortec 672) filtered the E stage signals with a shaping time of about 2 µs, corresponding to an electronic noise of about 3 keV FWHM-Si.

Fig. 2 shows a scheme of the acquisition system. The pulses generated in the two different stages, whose amplitudes are proportional to the energy deposited in the ΔE and E stages, were acquired by a custom multi-parameter multi-channel analyzer (mMCA) capable of processing the two signals event-by-event according to the selected coincidence mode. The mMCA was entirely realized by the laboratory of nuclear electronics of the Nuclear Engineering Department (Milano Polytechnic). It was optimized for the acquisition of signals generated in the monolithic silicon telescope.

![Fig. 2 Scheme of the electronic chain and acquisition system](image-url)
The mMCA was set-up so that the ΔE stage acts as a “master” detector for the E stage. The signals generated in the E stage are registered together with those of ΔE stage only when the ΔE stage itself triggers the coincidence. This permits to enable the acquisition of the time-correlated ΔE-E distribution whenever the interacting particle deposits in the ΔE stage an amount of energy higher than a fixed threshold. Since the ΔE stage measures a LET-related quantity, the acquisition events corresponding to low-LET particles are excluded through this threshold. Moreover, by triggering over the pulses generated in ΔE stage, all those signals generated in the E stage only are discarded. This permits to avoid the acquisition of events that do not cross the ΔE stage, which surely do not correspond to recoil-protons generated in the polyethylene. Therefore, this acquisition mode allows to realize a first particle discrimination on-line.

VI.2. Irradiations with monoenergetic neutrons

The experimental characterization of the detection system was carried out acquiring the responses of the two stages to monoenergetic neutrons. The irradiation fields were produced at the Van De Graaff accelerator of the INFN Legnaro National Laboratories (LNL, Legnaro). Neutrons were generated via the \(^7\text{Li}(p,n)^7\text{Be}\) reaction (threshold energy, \(E_{\text{th}}=1.88\text{ MeV}\)) by protons striking a thin LiF target (thickness 1 mg cm\(^{-2}\)) deposited on a carbon backing. The average current of the proton beam was measured with a Faraday cup connected to a charge integrator. The neutron energy and fluence produced via \(^7\text{Li}(p,n)\) reaction were calculated at the irradiation position with the DROSG-2000 code [55], by taking into account neutron production from the 0.430 MeV excited level of \(^7\text{Be}\).

The detector was placed at a distance of about 2.4 ± 0.1 cm from the target in order to have an adequate count rate (of the order of tens of counts per second) without worsening the angular resolution (about 1.2° at 2.4 cm).

The signals generated in ΔE and E stages are thus collected in pairs and the ΔE pulses act as a veto for the acquisition. At the end of each measurement, a matrix characterized by two columns and \(N_c\) rows, where \(N_c\) is the number of counts generated in the ΔE stage, is registered. Each column corresponds to a spectrum of events generated in each stage of the monolithic telescope and, after calibration, permits to evaluate their response function through a one dimensional binning. This procedure allows to obtain results analogous to those which would have been acquired with two independent multi-channel analyzers. Nevertheless, the coincidence technique permits to maintain the time-correlation between the ΔE and the E
signals at each event in the rows of the acquired matrix. Then, by means of a two dimensional binning, the so called \( \Delta E-E \) scatter plot can be constructed and the double differential distribution of energy deposited in the two telescope stages can be analyzed.

The scatter plots of the energy deposited in the \( \Delta E \) stage (\( E_d^{\Delta E} \)) against that deposited in the E stage (\( E_d^E \)) are shown in fig. 3 for various neutron energies. The distributions are normalized to the unit neutron fluence. Two different contributions can be observed at each neutron energy accounted for: one at high \( E_d^{\Delta E} \) values, due to recoil-protons, and another one at low \( E_d^{\Delta E} \) values, corresponding to secondary electron events. These electrons are due to photons generated: i) in the LiF target via the reactions \(^{19}\text{F}(p,\gamma)^{20}\text{Ne}, ^{19}\text{F}(p,\alpha\gamma)^{16}\text{O}\) and \(^7\text{Li}(p,\gamma)^8\text{Be} \) (\( Q \)-values +12.8 MeV, +8.1 MeV and +17.2 MeV, respectively) and ii) in the carbon backing by proton inelastic scattering on carbon (in this case the photons which affect mainly the measured spectra come from the 4.439 MeV \(^{12}\text{C} \) excited level to ground) [56-58].

![Fig. 3 Scatter plots of the energy deposited in the \( \Delta E \) stage (\( E_d^{\Delta E} \)) against that deposited in the E stage (\( E_d^E \)) for various neutron energies \( E_n \).](image-url)
Even if the $\Delta E$ stage is very thin, its planar extension is of the order of a few $\text{mm}^2$. Therefore, events related to electrons traveling not perpendicularly to the detector surface can deposit enough energy in the $\Delta E$ stage to exceed the acquisition threshold. The corresponding signals are then acquired and give rise to a low-LET contribution. However, the two populations of particles result to be well separated in the scatter plot. A further discrimination can be carried out off-line thus allowing to extract only the distribution of recoil-protons.

It should be underlined that this "two-step discrimination" (the first step is performed on-line, basing on the acquisition logic and the second one off-line, starting from the acquired scatter-plot) allows to separate with a high efficiency all crossers (i.e. the electrons and recoil-protons traversing the $\Delta E$ stage). Fig. 4 shows the E-stage spectrum acquired at 1.715 MeV neutron energy without any discrimination. The same spectra acquired with the $\Delta E$-E coincidence and further purged with the off-line discrimination are also shown in the same figure. It should be noted that the coincidence mode eliminates a lot of counts due to secondary electrons. The latter are orders of magnitude higher then the counts of recoil-protons at low-energies. In a second step, the off-line procedure permits to further improve the discrimination and to individuate the recoil-proton distribution over the entire range of deposited energy.

![Graph](image.png)

**Fig. 4** E-stage spectrum acquired at 1.715 MeV neutron energy without coincidence technique (black), by adopting a $\Delta E$ - E coincidence (blue) and after the off-line discrimination on scatter plot shown in fig.3 (red).
The discrimination threshold corresponds to the energy required by a recoil-proton for crossing the boundary between the $\Delta E$ and the $E$ stage, i.e. the maximum energy of the stoppers in the $\Delta E$ stage. This energy threshold depends on the angle at which a recoil-proton crosses the $\Delta E$-$E$ stage boundary (see figs. in chapter III). Moreover, by considering the relation between the maximum value of this angle $\Theta_{\text{lim}}$ (i.e. the angle at which a recoil-proton is able to cross the boundary for a given energy of the scattering neutron, referred as “crossing-angle” in the following) and the neutron energy, the threshold also depends on the neutron energy itself (fig. 5a). The dependence of the discrimination energy threshold on the crossing angle is shown more clearly in fig 5b for different neutron energies.

![Fig.5](image_url) (a) Scattering geometry at different neutron energies. (b) Dependence of the discrimination energy threshold on the crossing angle $\theta$ for different neutron energies.
This functions were obtained by exploiting the analytical model described in chapter III. Since the higher is the crossing angle, the higher is the energy threshold, the reference value for the lower energy limit for discrimination is assumed to be the worst case for each neutron energy. Since the total thickness of the monolithic silicon telescope is about 500 µm, the maximum detectable energy of the detection system is about 8 MeV. Thus, from fig. 5b, the theoretical discrimination limit for \( E_n = 8 \) MeV can be estimated to be about 615 keV, which corresponds to the energy below which the recoil-protons generated by 8 MeV neutrons can stop completely in the \( \Delta E \)-stage. This value is much higher than 300 keV, the energy threshold evaluated by the experimental scatter plot shown in fig. 3. This discrepancy can be explained by observing the actual probability distribution of the energy deposited by stoppers, already discussed in chapter III. In fact, only the 0.3 % of them deposit an amount of energy higher than about 350 keV. Thus, neglecting this stopper component, the operative energy threshold for discrimination results to be about 350 keV. It should be emphasized that this limit is of a factor 4 lower than that obtained in the previous works [11].

The response functions of the E stage at different neutron energies can be evaluated by elaborating the corresponding column of the matrix of the acquired events. The results of this

![Graph](image-url)  

**Fig. 6** Results of this one-dimensional binning of E stage counts normalized to the unit neutron fluence, together with the E stage response functions calculated analytically (chapter III).
one-dimensional binning, normalized to the unit neutron fluence, are shown in fig. 6, together with the E stage response functions calculated analytically (chapter III). The analytical curves were calculated by taking into account also the neutron production from the 0.430 MeV excited level of $^7$Be. The agreement is good at any neutron energy accounted for. The discrepancies at 0.680 MeV and 0.996 MeV neutrons were attributed to the poor counting statistics. However, it should be concluded that the analytical model is able to reproduce the E stage response with a satisfactory accuracy.

VI.3. Unfolding algorithm

An iterative unfolding algorithm based on the non-linear least-squared method was employed in order to reconstruct the primary neutron spectra from the measurement of the recoil-proton distributions. The response matrix was composed by the set of analytical response functions of the E stage $p_j(E_d^E)$ ($j = 0..N$) (chapter III) determined for N different values of $E_n$. For a quantitative evaluation, the functions $p_j(E_d^E)$ were multiplied by the corresponding probability $\pi_j(E_n)$ of generating a recoil-proton in the converter per unit neutron fluence, given by eq. 1. Starting from a distribution uniform in energy, the current spectrum is modified in each step in order to reduce progressively the $\chi^2$ of the data up to a value comparable with the degree of freedom of the problem.

The experimental $E_d^E$ spectra relating to the irradiations with monoenergetic neutrons

![Graph](image-url)  

**Fig. 7** Test of the unfolding algorithm. Neutron yield obtained by processing the response functions of the E stage (fig. 6)
were used to test the unfolding algorithm. The neutron yield obtained by processing the response functions of the E stage (fig. 6) are shown in fig. 7. The unfolded spectrum corresponding to 2.727 MeV neutron energy (blue line in fig. 7) shows a peak at about 2.3 MeV. This is due to 2.277 MeV neutrons coming from the 0.430 MeV excited level of $^7$Be. The FWHM of the unfolded peaks resulted to be about 30, 40, 120, 180 keV at 0.680, 0.996, 1.715 and 2.727 MeV, respectively. Even if these widths include other various sources of uncertainty, i.e. the spread of the neutron beams, the energy resolution of the E stage and the uncertainties of the analytical response matrix, they can be considered an estimation of the energy resolution of such a spectrometer. As discussed in chapter V, the E stage of the silicon telescope is characterized by an energy resolution of about 4 keV FWHM-Si. It should be underlined that the effective resolution of the neutron spectrometer is mainly affected by energy straggling in the radiator, in the dead layer and in the $\Delta E$ stage, which partially smear the advantages linked to the use of a silicon detector.

VI.4. Continuous neutron spectra

The feasibility of a neutron spectrometer based on a monolithic silicon telescope is demonstrated by its capability of resolving neutron spectra with continuous energy distribution. To this aim, the detection system was irradiated at the LNL with secondary neutrons generated at 0° by protons of different energies striking a thick beryllium target. At a given proton energy, neutrons are produced on beryllium via the reactions $^9$Be(p,np)$^{2\alpha}$, $^9$Be(p,np)$^8$Be, $^9$Be(p,n)$^9$B and $^9$Be(p, n$\alpha$)$^5$Li ($E_{th}$= 1.75, 1.85, 2.06 and 3.93 MeV, respectively).

The distance between the target and the detector surface was 4.3 ± 0.1 cm, corresponding to an angular resolution of about 0.7°. The $\Delta E – E$ scatter plots of the secondary radiation field were acquired by collecting the signals generated in the two stages of the detector in coincidence mode, as explained in the previous section. As an example, the spectrum of deposited energy in silicon obtained by bombarding the beryllium target with 5 MeV protons is shown in fig. 8. The high-LET contribution is due to recoil-protons, while the low-LET one may be ascribed to secondary electrons generated in the detector assembly by photons associated to the neutron field. These photons are mainly due to the de-excitation of $^{10}$B and of the excited states of $^9$Be, which is produced via proton inelastic scattering [58,59]. The separation between these contributions at different LET values permits to carry out a
good discrimination and to extract the spectrum of the energy deposited by recoil-protons only.

A folding algorithm was applied in order to verify the measured recoil-proton distribution. The expected distribution of the energy deposited in the E stage of the silicon telescope was calculated by multiplying the vector of spectral fluence measured with time-of-flight technique [60] by the analytical response matrix of the E stage. The result of this check is

![Graph showing the energy deposited in the E stage versus the energy deposited in the DeltaE stage.](image1.png)

**Fig. 8** Scatter plot of the energy $E_d^{\Delta E}$ deposited in $\Delta E$ stage versus energy $E_d^{E}$ deposited in E stage. The distribution was obtained by bombarding the beryllium target with 5 MeV protons.

A folding algorithm was applied in order to verify the measured recoil-proton distribution. The expected distribution of the energy deposited in the E stage of the silicon telescope was calculated by multiplying the vector of spectral fluence measured with time-of-flight technique [60] by the analytical response matrix of the E stage. The result of this check is

![Graph showing the response per unit charge versus energy deposited in the E stage.](image2.png)

**Fig. 9** Check of the unfolding procedure. Comparison between the experimental response of the E stage and folding of time-of-flight measurements [60]. Neutrons were produced at 0° by 5 MeV protons bombarding a thick beryllium.
shown in fig. 9 for a bombarding proton energy of 5 MeV. The agreement between the calculated and the measured spectra is good.

The energy distributions of the neutron yield were reconstructed with the unfolding algorithm. The results are compared in figs. 10-14 with data measured with time-of-flight techniques [60].

---

**Fig. 10** Energy distribution of the yield of neutrons generated at 0° by 3 MeV protons striking a thick beryllium target, measured with the diode spectrometer and TOF techniques [60].

**Fig. 11** Energy distribution of the yield of neutrons generated at 0° by 3.4 MeV protons striking a thick beryllium target, measured with the diode spectrometer and TOF techniques [60].
Fig. 12 Energy distribution of the yield of neutrons generated at 0° by 3.7 MeV protons striking a thick beryllium target, measured with the diode spectrometer and TOF techniques [60].

Fig. 13 Energy distribution of the yield of neutrons generated at 0° by 4 MeV protons striking a thick beryllium target, measured with the diode spectrometer and TOF techniques [60].
The spectra were generated by 3, 3.4, 3.7, 4.0 and 5.0 MeV protons striking a 1 mm thick beryllium target. The statistical uncertainty was calculated through a sensitivity analysis, by sampling the corresponding experimental pulse height distributions within their standard deviation. The contribution of the uncertainty related to the irradiation position was also accounted for. The global agreement between the resulting unfolded spectra and the reference data is fairly good above 400 keV, which approximately corresponds to the lower discrimination limit derived from $\Delta E-E$ scatter plots. For 3 MeV protons (figure 10), the distribution of the neutron yield measured with the silicon telescope reproduce only qualitatively the reference data at the lower neutron energies (below about 800 keV) probably because of the poor counting statistics of the corresponding experimental pulse height distribution. At higher bombarding proton energies the results are in agreement within the uncertainties at any neutron energy. Further measurements will be performed to better characterize the influence of the counting efficiency of the neutron spectrometer based on the monolithic silicon telescope.

It should be underlined that the neutron spectrometer based on the silicon telescope cannot reproduce the fine structure of the spectrum profile, while the time-of-flight technique provides an accurate description of the spectral distribution. Nevertheless, while the former is based on a simple detection system and can provide the global behavior of the spectrum
starting from a single measurement, the latter needs a complex apparatus and a systematic evaluation of neutron yield at each neutron energy.

The spectra measured at 3 and 3.4 MeV proton energy cannot be evaluated with a neutron spectrometer based on a simple photodiode, even with a pulse shape discrimination technique [10,11]. This is a confirmation of the capability of the monolithic silicon telescope of discriminating between recoil-protons and the photon background at low energies.

Table 1 lists the total neutron yields above 350 keV calculated by integrating the spectra shown in figs. 10-14, together with those measured in ref. 60. The results are in agreement within the uncertainties at any energy accounted for.

<table>
<thead>
<tr>
<th>Proton bombarding energy (MeV)</th>
<th>Angle</th>
<th>Neutron yield above 0.4 MeV (µC⁻¹ sr⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>This work</td>
</tr>
<tr>
<td>3</td>
<td>0°</td>
<td>(2.24 ± 0.46) × 10⁷</td>
</tr>
<tr>
<td>3.4</td>
<td>0°</td>
<td>(4.41 ± 0.83) × 10⁷</td>
</tr>
<tr>
<td>3.7</td>
<td>0°</td>
<td>(6.31 ± 1.17) × 10⁷</td>
</tr>
<tr>
<td>4</td>
<td>0°</td>
<td>(1.02 ± 0.15) × 10⁸</td>
</tr>
<tr>
<td>5.0</td>
<td>0°</td>
<td>(3.98 ± 0.39) × 10⁸</td>
</tr>
</tbody>
</table>

**Table 1** - Total (> 0.4 MeV) yield of neutrons generated by protons of several energies striking a thick beryllium target.

It should be underlined that the lower detectable energy of about 350 keV can be further reduced by exploiting the ΔE-stage spectra of stoppers. These distributions can be easily calculated from the acquisition matrix by extracting only those rows for which the corresponding amplitude of the E stage signal is 0. This elaboration could allow to decrease the detection limit, given by the contribution of low-LET particles to the spectra measured by the thin ΔE stage. However, several problems related to the addition of the spectra acquired with the two stages has still to be solved and the true value of the minimum detectable energy of such a neutron spectrometer is presently under investigation.
Solid-state silicon detectors are challenging devices for microdosimetry, mainly because they can provide sensitive zones (i.e. depletion layers) of the order of a micrometer. This allows to measure physical events in a real micrometric site differently from tissue equivalent proportional counters (TEPCs), where the site is simulated acting on the gas pressure[]. Moreover, silicon detectors are characterized by a high spatial and a good energy resolution. However, feasibility studies concerning the application of such detectors for microdosimetry (chapter II) emphasized several problems, mainly the field-funnelling effect (due to a local distortion of the electric field in the depletion layer, induced by high-LET particles, leading to the collection of electron-hole pairs produced in the non-depleted zone), the non tissue-equivalence of silicon and the geometry of the sensitive volume, which is usually parallelepiped (and not spherical like in the conventional microdosimeters, such as the tissue equivalent proportional counters).

This chapter focuses on the study of a microdosimeter based on a monolithic silicon telescope, consisting of a ΔE and an E stage-detector, about 1.9 μm and 500 μm thick, respectively. The ΔE stage is in principle a silicon microdosimeter when coupled to a tissue-equivalent (TE) converter. The E stage can provide information on the energy and the type of the incident radiation and could be useful in the optimization of tissue-equivalence and shape equivalence corrections. The attention was focused on this device, since the field-funnelling effect was expected to be negligible or absent, as demonstrated by the characterization of this detector through irradiations with alpha particles (chapter V).

Irradiations with monoenergetic neutrons were performed at the INFN-Laboratori Nazionali di Legnaro (Legnaro, Italy) in order to verify the possibility of employing the monolithic silicon telescope for microdosimetry of neutron fields. The preliminary results of an analytical approach for the correction of a spectrum measured with this silicon-based
microdosimeter for tissue-equivalence and geometrical effects will be also presented. The non
tissue-equivalence of silicon was corrected exploiting the signals generated in the E-stage.
The correction for the sensitive volume geometry was optimized taking into account the track
length distribution of the recoil-protons generated in the converter (chapter IV). The derived
do.se distribution of the energy imparted per event was compared with that measured with a
cylindrical TEPC.

**VII.1 Main features of the detector**

The main features of the silicon telescope which was investigated as a microdosimeter
will be recalled in this section for better evidence. The monolithic silicon telescope (figure 1)
consists of a $\Delta E$ and an E stage-detector, 1.9 $\mu$m and 500 $\mu$m in thickness, respectively. The
sensitive area of the detector is 1 mm$^2$. The two stage-detectors are fabricated on a single
silicon substrate and share the p$^+$ cathode which is realized through a deep ion implantation.
The charges generated in silicon by the incident radiation are collected separately by the two
stage-detectors and the highly-doped p$^+$ cathode acts as a “watershed” for this separation. The
$\Delta E$ stage is in principle a silicon microdosimeter when coupled to a TE converter. The E stage
can provide information on the energy and the type of the incident radiation.

![Fig. 1 Sketch of the ST-Microelectronics Monolithic Silicon Telescope](image)

The capacitance of the $\Delta E$ stage (about 165 pF) leads to employ a low-noise
preamplifier with a high-capacitance input JFET. Presently, the minimum detectable energy is
around 8 keV, using a commercial preamplifier (Amptek A250) connected to an amplifier
(ORTEC 672) with a shaping time of 2 $\mu$s.
The silicon detector was coupled with a polyethylene converter generating recoil-protons through elastic neutron scattering. The contribution of recoil carbon ions was very small at the neutron energies accounted for. Only recoil-protons were taken into account in this study for simplicity. Tissue-equivalent converters will be considered in a second step of the present research.

**VII.2 Field Funneling Effect**

The silicon telescope, coupled to a polyethylene converter, was irradiated with monoenergetic neutrons produced at the Van De Graaff accelerator of the INFN-LNL. Monoenergetic neutrons were produced by bombarding a thin LiF target (i.e. via the reaction $^7\text{Li}(p,n)^7\text{Be}$) with accelerated protons. Only the $\Delta E$ stage was inversely biased during these measurements. The spectra of energy deposited in the $\Delta E$ stage are shown in figure 2 for different energies of the incident neutrons. As expected, the mean energy of recoil-protons decreases with increasing neutron energy. The spectra were not observed to shift like those acquired with the n-p diode matrix described in chapter II and shown in fig. 3. Therefore, the field funnelling effect (FF) should be negligible or even absent. The small variations of the maximum energy of the spectra were ascribed to geometrical effects due to the direction of incidence of the recoil-protons on the $\Delta E$ stage.

![Fig. 2 Spectra of energy imparted ε in the ΔE stage for different energies $E_n$ of the incident neutrons. The edge at higher ε values does not depend on the neutron energy. Therefore the “field funnelling effect” should be negligible.](image)
Chapter VII       A Solid State Microdosimeter based on a Monolithic Silicon Telescope

The validity of this assumption was investigated with Monte Carlo simulations with the FLUKA code [36-39]. Obviously, the FF is not treated by the simulation code and therefore an agreement of the simulated data with the experimental ones should enforce the hypothesis made above. The geometry of the detector was approximated with a wafer of three parallelepipeds simulating the dead layer (0.2 µm thick), the $\Delta E$ (1.9 µm thick) and the E (500 µm thick) stage. The dead layer was placed in contact with a 1 mm thick polyethylene converter. The source was a parallel beam of monoenergetic neutrons. The transport of secondary electrons generated by recoil-protons in silicon through ionisation was considered. The energy resolution of the detector was not accounted for. Photons associated to the neutron field, generated mainly from the de-excitation of $^{20}$Ne which is produced via the reaction $^{19}$F(p,n)$^{20}$Ne [56-58], were not taken into account in the simulations.

The response functions of the $\Delta E$ stage were calculated also with the analytical model described in chapter III and validated with FLUKA simulations. The experimental data of the energy deposited in the $\Delta E$ stage by recoil-protons from 0.680, 0.996, 1.306 and 1.715 MeV neutrons are shown in figure 4 together with the spectra obtained by simulations and analytically. The agreement with the experimental data is satisfactory at any neutron energy accounted for. In particular, the edge of the simulated and analytical spectra are superimposed to the experimental ones, confirming that the FF is negligible. Therefore, the small variations of the maximum energy of the measured spectra at different neutron energies were ascribed to

![Fig. 3 Results of the irradiation with monoenergetic neutrons of energy $E_n$ of a device consisting of a matrix of nine n-p diodes (sensitive area of 1 mm$^2$) which was realized through the AMS BiCMOS 0.8 µm technology.](image-url)
geometrical effects related to the track-length distribution of the recoil-protons (described in chapter IV).

![Experimental data of the energy deposited in the ΔE stage by recoil-protons from neutrons of different energies together with the spectra obtained by simulations (FLUKA code) and analytically (model described in chapter III).](image)

**Fig. 4** Experimental data of the energy deposited in the ΔE stage by recoil-protons from neutrons of different energies together with the spectra obtained by simulations (FLUKA code) and analytically (model described in chapter III).

### VII.3 Monolithic Silicon Telescope as a Microdosimeter

The experimental spectra acquired with the ΔE-detector relate to the energy imparted by secondaries generated in the converter in a sensitive volume of silicon, while the reference microdosimetric spectra always refer to tissue volumes. Therefore, a proper correction of the “non tissue-equivalence” of silicon with respect to recoil-protons is needed in order to compare the spectra measured by the two different detection systems. An analytical method based on the range-energy relationship (chapter III) was assessed for this purpose. Moreover, the profiles of these measured distributions are strictly related to the shape of the sensitive volume through the track length distributions. Thus, an analytical correction for the shape-equivalence is also required.
Chapter VII  

A Solid State Microdosimeter based on a Monolithic Silicon Telescope

The difference between silicon and soft-tissue in terms of energy deposited along a track of a fixed length depends on the particle energy. The latter information can be obtained by acquiring also the signal generated in the E detector (thickness 500 µm). An optimized correction function accounting for the energy-dependence of tissue-equivalence was calculated and tested. The derived spectra were compared with a microdosimetric spectrum measured using a cylindrical TEPC filled with propane tissue-equivalent (TE) gas. The simulated dimensions (height and diameter) of the TEPC were equal to 2.67 µm.

The silicon microdosimeter here discussed shows many limitations, mainly the high value of the minimum detectable energy (due to the electronic noise) and a planar extension of the order of 1 mm² (i.e. a wide sensitive area). In spite of these constraints, it shows interesting features from the point of view of cheapness, simplicity and transportability. The main limitations and advantages of silicon microdosimeters are discussed in details in ref. [6].

The following sections discuss the main limiting factors of a silicon microdosimeter and the analytical corrections which were developed for the response of the monolithic telescope to monoenergetic neutrons.

VII.4 Experimental

The monolithic telescope, coupled to a polyethylene converter, was irradiated with monoenergetic neutrons produced at the LNL-Van De Graaff. Both the ΔE and the E-detectors were biased and the signals generated in the two stages were amplified and shaped using two independent electronic chains. The electronic pulses generated in the two stages were acquired by a two-channel ADC in coincidence mode, in order to discriminate recoil-protons from secondary electrons generated by background photons.

The minimum detectable energy of the ΔE stage is limited to about 10 keV by the electronic noise.

The ΔE-E scatter plot obtained by irradiating the silicon telescope with 2.7 MeV monoenergetic neutrons is shown in figure 5. The high-LET distribution of recoil-protons is well separated from the low-LET one, which is ascribable to secondary electrons generated in the detector assembly by photons associated to the neutron field. The two contributions are well-discriminated and it is possible to eliminate the contribution of secondary electrons in order to obtain the spectrum of energy imparted in the ΔE detector by recoil-protons only.
VII.5 Tissue-equivalence correction

As already mentioned, the solid state microdosimeter was realized by coupling a 1 mm thick polyethylene layer to the monolithic silicon telescope. When exposed to a neutron field, the silicon device acts as a detector of the energy deposited in the sensitive zone by the secondary charged particles generated in the polyethylene converter.

For the “silicon-to-tissue” corrections discussed below, an important requirement is that the contribution of secondaries generated by direct interactions of neutrons with silicon is negligible. In other words, most events should be crossers or stoppers.

The most important reactions that can generate starters or insiders in silicon devices are: i) the elastic scattering of neutrons with silicon nuclei, that can produce recoils with energy up to the 13% of that of the impinging neutrons; this contribution was estimated to be about one order of magnitude lower than the one due to recoil-protons generated by neutrons above 1 MeV; ii) the $^{28}\text{Si}(n, \alpha)^{25}\text{Mg}$ and $^{28}\text{Si}(n,p)^{28}\text{Al}$ reactions (with threshold energies of 2.75 MeV and 4 MeV, respectively), whose contribution was estimated in ref. 10 and resulted to be one order of magnitude lower than that of recoil-protons at the neutron energies in the MeV range.; iii) the $^{10}\text{B}(n, \alpha)^{7}\text{Li}$ reaction (thermal neutron cross-section $\cong 3800$ b) that can be promoted by neutron capture on boron, which is the p-type dopant of the silicon telescope; this contribution was avoided by implanting pure $^{11}\text{B}$.  

Fig. 5 The $\Delta E$-$E$ scatter plot obtained by irradiating the silicon telescope with 2.7 MeV monoenergetic neutrons.
Since these contributions have demonstrated to be negligible (of a few percent at maximum, depending on the neutron energy), it can be assumed that the spectrum of energy deposited in the silicon ΔE-detector is only due to secondary particles generated in the converter. A correction must be applied to the measured deposited energy in order to reproduce tissue-equivalence conditions, i.e. in order to obtain a spectrum equivalent to that measured with an hypothetical tissue ΔE-detector. This correction is based on the analytical model described in chapter III.

The energy $E_d^{Si}(E_p,s)$ deposited by recoil-protons of energy $E_p$ in the silicon ΔE-detector was calculated for a fixed track of length $s$. The corresponding energy deposited along the same track length in a tissue-equivalent ΔE-detector $E_d^{Tissue}(E_p,s)$ can be calculated by scaling the energy $E_d^{Si}(E_p,s)$ with the following expression:

$$E_d^{Tissue}(E_p,s) = E_d^{Si}(E_p,s) \cdot \frac{S^{Tissue}(E_p)}{S^{Si}(E_p)} \quad (1)$$

where $S^{Si}(E_p)$ and $S^{Tissue}(E_p)$ are the stopping powers for protons of energy $E_p$ in silicon and in tissue, respectively.

Fig. 6 Calculated deposited energy in a track of length $s$ in silicon (black line) and in tissue (red line) at different proton energies.
Figure 6 shows the calculated energy deposited in silicon and in tissue along tracks of various lengths for different proton energies. Figure 7 shows the results of the scaling procedure for the tracks lengths mentioned above, through which it is possible to correct the measured deposited energy for tissue-equivalence.

It should be remembered that the values of the stopping powers $S^S(E_p)$ and $S^{\text{Tissue}}(E_p)$ were taken from ICRU report n. 49 [34]. These corrections are more than satisfactory for all the different track lengths considered (figure 7). The tissue-equivalence scaling factor depends obviously on the energy of the recoil-proton impinging on the detector, as was accounted for in expression (1). This energy can be measured event-by-event by exploiting the E-stage of the monolithic silicon telescope. This is an important feature which justifies the use of a telescope detector for silicon microdosimetry. Figure 8 shows the dose distributions of the energy imparted per event measured with the $\Delta E$ detector before and after the analytical correction for tissue-equivalence.

The validity of this procedure was investigated with FLUKA simulations. A tissue-equivalent detector with the same geometrical structure of the silicon monolithic telescope was modelled. The geometry was approximated with a wafer of three parallelepipeds simulating the dead layer (0.2 $\mu$m thick, in titanium), the $\Delta E$ (1.9 $\mu$m thick) and the E (500 $\mu$m thick) stage. The dead layer was placed in contact with a 1 mm thick polyethylene
converter. The source was a parallel beam of 2.7 MeV neutrons. Secondary electrons generated by recoil-protons were transported and energy straggling was accounted for. The statistical uncertainty of the simulated spectrum is lower than 5% in most of the considered intervals of deposited energy. The simulated spectrum was compared with the experimental

![Fig. 8](image_url)  
**Fig. 8** Dose distribution of the energy imparted per event in the $\Delta E$ stage before (black curve) and after (orange curve) the correction for tissue-equivalence. The correction is based on equation 1.

![Fig. 9](image_url)  
**Fig. 9** Dose distribution of the energy imparted per event in the $\Delta E$ stage by recoil-protons generated in the converter corrected for tissue-equivalence (black curve), compared to the FLUKA simulation of the energy deposited in an hypothetical tissue detector with the same geometrical structure of $\Delta E$ stage (violet curve).
one, corrected with the procedure based on expression (1). The results are shown in figure 9. The agreement between the two spectra is satisfactory. The discrepancies at low energies (in the interval 20-40 keV) and in the proton edge (around 170 keV) may be due to the approximations of the simulated geometry.

**VII.6 Shape analysis and correction criteria**

The sensitive volume of the ΔE-detector can be approximated to a rectangular parallelepiped.

The track length distribution of recoil-protons is related to the geometry of both the interaction and the detector. In order to compare the spectra measured with the silicon microdosimeter and with a cylindrical TEPC, it is necessary to adopt some criteria for choosing the dimension h (i.e. the height and the diameter) of the simulated tissue right cylinder. Figure 10 shows the track length distribution of the silicon microdosimeter irradiated with 2.7 MeV neutrons together with that of the cylindrical TEPC. The optimum cylindrical dimension is calculated by equating the dose-mean energy imparted per event $\bar{\varepsilon}_D$, following the parametric criteria for the equivalence of shapes given in the literature [21,40,61] and already described in chapter IV, section 4. Therefore, assuming a constant energy transfer rate $L$, $\bar{\varepsilon}_D$ is given by:

$$\bar{\varepsilon}_D = L \cdot \frac{\int_0^\infty s^2 \cdot p(s) ds}{\bar{s}}$$

(2)

where $p(s)$ is the probability density distribution of a track of length $s$ and $\bar{s}$ is the mean value of $p(s)$. By equating the $\bar{\varepsilon}_D$ for the two different shapes considered, i.e. for the two track length distributions shown in figure 10, it is possible to evaluate the optimum dimension of the equivalent right cylinder in terms of the dose-mean energy imparted per event. In the case of a 1.9 µm thick ΔE-detector, this method estimates the optimum cylinder dimension at about 2.67 µm.

It should be underlined that this evaluation is valid for neutron energies above 1 MeV, where this shape correction factor shows a slight variation (below 3%) with the energy of the impinging neutrons. This behaviour can be observed in figure 8 of chapter IV, where the second factor in the right member of eq. (2) was calculated for the two different volumes at different neutron energies.
VII.7 Inter-comparison with a TEPC

A cylindrical TEPC filled with a propane-based tissue-equivalent gas was irradiated with monoenergetic neutrons at the LNL Van De Graaff accelerator, under the same conditions selected for measuring the ΔE-stage spectrum shown in figure 2. The pressure of the filling gas was set in order to simulate a 2.67 μm cylindrical site, according to the condition derived from expression (2).

The dose distributions of the energy imparted per event acquired with the TEPC and with the silicon telescope after the tissue-equivalence correction are compared in figure 13. The agreement is rather satisfactory. The calculated frequency-mean energy imparted per event $\bar{\varepsilon}_f$ results to be 66.8 ± 2.7 keV for the cylindrical TEPC and 71.6 ± 4.3 keV for the silicon microdosimeter, while the dose-mean energy imparted per event $\bar{\varepsilon}_D$ is equal to 104.2 ± 4.2 keV for the TEPC and 105.5 ± 6.3 keV for the silicon microdosimeter. The discrepancies observed in the dose distributions shown in figure 13 at the lower-energies and in the proton edge may be due to geometrical effects: despite the optimization for shape-equivalence, the inherent difference in the track length distribution still subsists.
Fig. 13 Inter-comparison between the dose distribution of the energy imparted per event acquired with a TEPC (2.67 µm site diameter) (red curve) and with ∆E stage of the silicon telescope after the corrections for tissue-equivalence and geometrical shapes (black curve). The detectors were irradiated with 2.7 MeV monoenergetic neutrons.
The use of silicon diodes for neutron spectrometry was limited, even with pulse-shape discrimination techniques, to a minimum detectable energy of about 1.5 MeV. This threshold was due to the contribution of secondary electrons (generated by photon background) to the spectra of recoil-protons. An alternative silicon device was investigated in order to develop a recoil-proton spectrometer capable of resolving neutron spectra at lower energies. The prototype proposed in the present work was the monolithic silicon telescope, a detector constituted by a 1.9 µm thick \( \Delta E \) stage and a 500 µm thick \( E \) stage. A polyethylene radiator of 1 mm in thickness was coupled to a device with a sensitive area of about 1 mm\(^2\). The intrinsic efficiency of the detection system was about \( 10^{-4} \) counts per unit neutron fluence. The silicon telescope was investigated both analytically and experimentally. In particular, a general model of the response functions of the telescope stages was developed and validated with Monte Carlo simulations and experimentally. By exploiting the coincidence of signals generated in both stages by the incident radiation, recoil-protons resulted to be well-discriminated from secondary electrons for energies above about 300 keV. The unfolded spectra have been compared with the results obtained from time-of-flight data taken from the literature. Some discrepancies mainly due to poor counting statistics were observed. However, the agreement was satisfactory for neutron energies above 300 keV, a factor of five lower than the minimum detectable energy of the neutron spectrometer based on a simple silicon diode. A further reduction of this limit could be achieved by taking into account the \( \Delta E \) stage spectra of stoppers. This issue will be the matter of a future research. The maximum detectable energy is related to the thickness of the \( E \) stage and is about 8 MeV for the present device.

The monolithic silicon telescope was also investigated and tested as a solid state microdosimeter. The sensitive thickness of the \( \Delta E \) stage of the telescope can be considered independent of the LET of the ionising particles (i.e. the field funnelling effect is negligible) and therefore it shows interesting features for measuring the distribution of the energy imparted to microscopic volumes. Owing to the electronic noise, the minimum detectable energy is about 10 keV. This limits the applicability of this silicon microdosimeter to high LET particles.
An optimised tissue-equivalence correction was carried out by exploiting the energy deposited by recoil-protons in the E stage. The method applied for shape-equivalence of the track distribution is limited to neutrons above 1 MeV, while the thickness of the E stage restricts the application of the procedure to neutrons below 8 MeV.

The preliminary results of an inter-comparison with a TEPC simulating a 2.67 μm cylindrical site in tissue have been discussed. The agreement between the dose distributions of the energy imparted per event was fairly satisfactory, but discrepancies due to geometrical effects still persist. A systematic comparison at different neutron energies must be performed in order to estimate the role of these effects in microdosimetric measurements of continuous neutron spectra. A telescope constituted by a matrix of cylindrical ΔE stages of a few tens of micrometers in sensitive area could be useful for minimizing these geometrical effects. This matrix has been recently manufactured by ST Microelectronics and will be tested later this year.

In conclusion, the monolithic silicon telescope demonstrated to be a versatile device, capable of measuring continuous neutron spectra in the energy interval (0.3 – 8.0 MeV) and microdosimetric spectra in the interval (1.0 – 8.0 MeV). It should be underlined that these results were obtained by means of simple, cheap and easy of use detection systems.
REFERENCES


[34.] International Commission on Radiation Units, Stopping Power and Ranges for Protons and Alpha Particles (Report 49), ICRU, Bethesda, MD, 1993.

[35.] JEF 2.2 Nuclear Data Library- JEFF Report 17, NEA Data Bank (April 2000).


[37.] A. Ferrari and P.R. Sala, The Physics of High Energy Reactions, Proceedings of the Workshop on Nuclear Reaction Data and Nuclear Reactors Physics, Design and Safety, International Centre for Theoretical...
References


[43.] A. Cauchy, Memoire sur la rectification des courbes et la quadrature des sourface courbe, Oeuvres Completes, Paris (1908)


[55.] M. Drosg, DROSG-2000, codes and database for 57 neutron source reactions, documented in the IAEA report IAEA-NDS-87 Rev. 7 (January 2002), received from the IAEA Nuclear Data Section.


